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THE PREDICTION OF THERMAL TRANSIENTS IN  
COMPACT HEAT EXCHANGERS. VOLUME 1:  
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COMPUTER PROGRAM FOR THE PREDICTION OF  
THERMAL TRANSIENTS IN COMPACT  
HEAT EXCHANGERS

VOLUME I - DEVELOPMENT OF THEORY AND  
SOLUTION PROCEDURES

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This work was carried out under the direction of D. M. Dix, with E. P. Demetri assuming project responsibility. M. Platt was the other major participant in this program.

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## SUMMARY

This is the first of two volumes dealing with the prediction of the dynamic thermal behavior of compact heat exchangers. This volume contains a presentation of the analytical methods derived for calculating the transient performance. The second volume provides a complete description of the digital-computer program (Program KRONOS) developed on the basis of these methods.

Models representing the transient heat-transfer processes occurring in compact heat exchangers of the plate-fin variety have been developed for single-phase and condensing fluids. Based on the assumed heat-transfer models, the differential equations governing the transient thermal performance have been derived for parallel-flow, counterflow, and multipass-crossflow configurations. An iterative finite-difference procedure has been developed for the solution of these governing equations. This procedure can be used to compute the variation of outlet conditions with time for each fluid in a given exchanger for specified initial temperature distributions and prescribed variations in inlet conditions and flow rate. The overall method developed for analyzing the transient behavior has proved quite effective in treating the types of compact heat exchangers most often encountered in aerospace applications.

## INTRODUCTION

### Background

A compact heat exchanger is seldom considered as a separate and distinct unit, but is instead always an integral part of a larger system. As such, it is often an important factor in determining the performance of the over-all system. In many aerospace applications, it is necessary to investigate the dynamic behavior of systems containing one or more heat exchangers where the characteristics of the heat exchangers significantly affect the over-all transient response to changes in the operating variables. In these instances, a reliable method of predicting the performance of a heat exchanger under transient conditions is required.

Models representing the transient heat-transfer processes occurring in compact heat exchangers of the plate-fin variety have been developed for two different categories of working fluids--common single-phase fluids and condensing fluids; the latter category consists of both single-component fluids and two-component fluids (for example, humid air) in which only the less volatile component condenses. No attempt has been made to consider two-phase boiling fluids, since this would introduce more complication than appears worthwhile at the present time in view of the approximate nature of the boiling heat-transfer data available. The models developed include the effect of the thermal capacitance of the side walls, which make up the outer shell separating the heat-exchanger core from the environment (see sketches included in the Nomenclature), on the transient response. This effect can be important, although it is usually neglected in more approximate treatments of the dynamic behavior of heat exchangers.

The differential equations governing the transient thermal performance have been derived, on the basis of the assumed heat-transfer models, for the heat-exchanger configurations most often encountered in aerospace systems; that is, parallel-flow, counterflow,



and multipass-crossflow arrangements of plate-fin matrices (heat-transfer surfaces). A procedure for solving the governing differential equations has been developed and incorporated into a digital-computer program (Program KRONOS) to facilitate the calculations involved in determining the transient thermal performance of a specified heat-exchanger configuration. The program can be used to calculate the variations of pressure drop and outlet conditions with time for given initial conditions and prescribed time variations in inlet temperature (and absolute humidity or vapor quality for a condensing fluid) and flow rate of both streams. The initial conditions are defined as the temperature (and vapor quality for a single-component condensing fluid) distributions in the exchanger immediately prior to the initiation of the transient. These can correspond to either a start-up or a steady-state condition.

#### Method of Attack

A fairly complete description and literature review of the currently available methods of heat-exchanger transient analysis are given in Chapter VII and Appendix IV of Reference 1. Closed-form solutions of the transient equations for parallel-flow and counter-flow exchangers are presented in the literature. These solutions are quite complicated and difficult to use for practical calculations despite the fact that they involve a variety of approximations (such as, assuming constant fluid properties and heat-transfer coefficients) in order to obtain the closed-form relations. There are no methods available for treating multipass-crossflow exchangers. The few results reported in the literature for single-pass crossflow are for the most part confined to a fairly narrow range of conditions. The majority of these procedures for any type of flow configuration are limited to cases where there is a step change in the inlet temperature or flow rate of one of the fluids in the exchanger. Since both the temperature and flow rate can change simultaneously, the procedures as they currently stand would have to be modified,

further increasing their complexity.

The analytical procedure chosen here for the calculation of the transient performance basically consists of a direct numerical solution of the governing differential equations for the types of working fluids and flow arrangements described previously. The approach used involves expressing the partial derivatives with respect to time in finite-difference form in the differential equations representing the heat-transfer processes occurring at any time ( $\theta$ ). For the case of single-phase fluids, this leads to a set of differential equations in which the only derivatives are those of the fluid temperatures with respect to flow length on both sides. By substitution, the number of equations in the set is reduced to two, one for each side of the exchanger. The two final working equations contain as the only independent variables: the spatial derivatives of the fluid temperatures, the local fluid temperatures at the current time ( $\theta$ ) and at the preceding time ( $\theta' = \theta - \Delta\theta$ ), and the local metal temperatures (of the separating wall and side walls) at the preceding time.

As part of the problem statement for any particular case, the temperature distributions throughout the exchanger at the start of the transient,  $\theta \equiv 0$ , are either given (start-up initial conditions) or are calculated from specified information (steady-state initial conditions). In addition, the inlet temperature of each fluid is prescribed as a function of time (boundary conditions). With this information, the two working equations are solved simultaneously to obtain the distributions of fluid temperature at each value of time ( $\theta$ ) desired. This is accomplished by integrating numerically the differential equation related to each fluid. The integration for each fluid is performed with respect to distance along its flow length from the inlet to the outlet. The solution procedure for each time  $\theta$  is iterative, due to the selected form of the equations and the desirability of allowing the fluid properties and heat-transfer coefficients to vary with temperature. Once the

fluid-temperature distributions at time  $\theta$  have been obtained, the corresponding metal temperatures are computed in a straightforward fashion using the pertinent equations from the original set. This procedure is carried out for each desired value of  $\theta$ , starting at  $\theta = 0$  and proceeding in a stepwise manner in increments of  $\Delta\theta$  until the desired time interval has elapsed. The method used in cases where one of the fluids is condensing is essentially the same as described here with minor variations in detail.

#### Report Arrangement

This report consists of two volumes. This first volume contains a presentation and discussion of the analytical methods and procedures on which the computer program is based. Volume II provides a complete description of the computer program, Program KRONOS.

The next section of this volume contains a presentation of the differential equations governing the transient heat-transfer processes. This is followed by a section in which the procedure for solving the governing equations and computing the transient performance is described. The last section of the main body contains a number of conclusions and recommendations related to the work described in this report. Finally, the pertinent details related to the derivations of the governing equations and solution procedure are presented in several appendices.

## GOVERNING EQUATIONS

This section of the report discusses the differential equations governing the transient heat-transfer processes occurring in compact heat exchangers of the type described in the previous section. The discussion begins with a presentation of the limiting assumptions made in deriving the equations. This is followed by a description of the manner in which the heat exchanger is modeled for purposes of analysis. Next, the governing equations are presented for the various flow arrangements and fluid types considered. Finally, the initial and boundary conditions which must be specified in order to solve the system of equations are presented and discussed.

### Limiting Assumptions

The derivations of the relations presented here involve a number of simplifying assumptions made in order to obtain a system of equations which can be solved in a practical and efficient manner. The major assumptions are listed as follows:

1. The heat exchanger is adiabatic. This implies that the exchanger is thermally insulated from the outside surroundings and all of the heat transfer occurring takes place within its boundaries.
2. Longitudinal conduction is negligible in both the fluids and the exchanger structure.
3. There is no scale deposition on any of the walls and the wall resistances themselves are negligible compared to the convective heat-transfer resistances.
4. The fluid temperature within each passage is constant in the nonflow directions.
5. The temperatures vary only in the flow directions. This means that, for parallel-flow and counterflow arrangements, the temperatures are functions of a single spatial coordinate (  $x$  ) and, for multipass crossflow, they are functions of the two spatial coordinates of

the flow plane (  $x$  and  $y$  ).

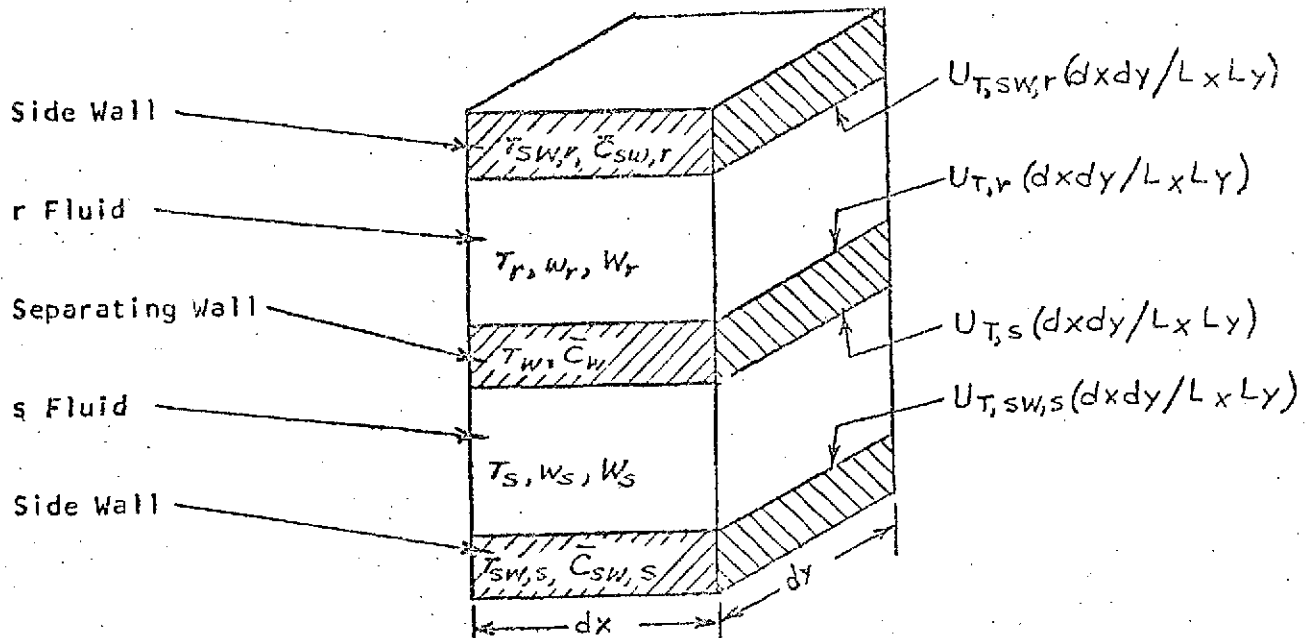
6. There is no lateral mixing in either of the two fluids.
7. The pressure of a condensing fluid is constant throughout the exchanger. This does not affect the accuracy of the relations to any great extent, since for a condensing fluid the pressure drop due to friction is somewhat compensated for by the pressure rise due to the density change associated with the process of condensation.
8. If condensation occurs, there is no accumulation of moisture in the heat exchanger; that is, the total mass flow rate leaving the exchanger is equal to that entering it at any point in time.

A number of additional assumptions are made in deriving the relations for a two-component fluid in which the less volatile component condenses (wet gas):

9. Liquid-vapor phase equilibrium exists at each point in the exchanger where liquid exists. This means that at any position in the exchanger where the fluid temperature is below the dew point, the fluid is saturated, such that the enthalpy and absolute humidity of the stream are both functions of temperature only.
10. All of the condensation occurs on the walls.
11. The heat capacity of the liquid film on the walls is negligible compared to the heat capacity of the walls themselves.
12. The heat-transfer coefficient,  $h$ , is the same as would be obtained for a single-phase fluid.

#### Representation of Heat Exchanger

For purposes of transient analysis, a heat exchanger can be considered to consist of a number of general differential elements such as shown schematically in the sketch on the following page.



Differential Element of Heat Exchanger

In all cases, one of the fluids (designated arbitrarily as the  $s$  fluid) must be single-phase throughout. The other fluid in the exchanger (designated as the  $r$  fluid) can be single-phase throughout, a single-component condensing fluid, or a two-component fluid from which the less volatile component is condensing (wet gas). For multipass-crossflow configurations, the  $s$  fluid must be the one which is turned between passes. This eliminates the possibility of turning the two-phase fluid, if there is one. One reason for this restriction is that there are no methods available for calculating the pressure drop associated with turning a two-phase fluid. Another and more important reason is that the turning may cause flow distortions due to the centrifuging of the liquid component which cannot be accounted for adequately in the analysis.

In the sketch of the differential element, the quantities,  $W_r$  and  $W_s$ , are the inventories of the two fluids in the heat exchanger,  $W = \rho V_{\text{void}}$ , where  $\rho$  is the local fluid density and  $V_{\text{void}}$  is the total void volume on one side of the

heat exchanger. The terms  $\bar{C}_w$ ,  $\bar{C}_{sw,r}$ , and  $\bar{C}_{sw,s}$  are the heat capacitances of the walls, each defined as the product of the specific heat and mass of the metal. The expressions for calculating  $V_{void}$  and  $\bar{C}$  are presented in Appendix I along with the relations for computing other pertinent geometric properties of the heat-exchanger core. The variable,  $U_T$ , represents the total thermal conductance for heat transfer between each fluid and the metal surfaces over which it flows; for example,  $U_{T,r} = \eta_{o,r} h_r A_r$  is the conductance for heat transfer between the  $r$  fluid and the separating wall and the fins on the  $r$  side attached to it. The conductances for each fluid are functions of the heat-transfer coefficient, surface area, and surface efficiency corresponding to that fluid. The procedures and relations involved in determining these quantities for the various types of fluids are discussed in Appendix II.

#### Transient Differential Equations

The equations are presented below for the three different cases corresponding to the different types of fluids. A set of five simultaneous differential equations is obtained for each case. These equations are derived by taking an energy balance for each component of the general differential element represented in the preceding sketch. For all flow arrangements, the flow direction of the  $r$  fluid is along the positive  $x$  direction of the flow plane (see sketches included with the Nomenclature). The flow direction of the  $s$  fluid is along the positive  $x$  direction for parallel flow, along the negative  $x$  direction for counterflow, and along the positive  $y$  direction for multipass crossflow.

Single-Phase Fluids on Both Sides

An energy balance on the r fluid in the element gives the following equation for all flow arrangements,

$$\begin{aligned} \left( \frac{W_r}{L_x L_y} c_{P,r} \right) \frac{\partial T_r}{\partial \theta} dx dy = & - \left( \frac{W_r}{L_y} c_{P,r} \right) \frac{\partial T_r}{\partial x} dx dy \\ & + \left( \frac{U_{T,r}}{L_x L_y} \right) (T_w - T_r) dx dy + \frac{U_{T,sw,r}}{L_x L_y} (T_{sw,r} - T_r) dx dy \end{aligned} \quad (1)$$

A similar energy balance on the s fluid for parallel-flow and counterflow arrangements gives,

$$\begin{aligned} \left( \frac{W_s}{L_x L_y} c_{P,s} \right) \frac{\partial T_s}{\partial \theta} dx dy = & \mp \left( \frac{W_s}{L_y} c_{P,s} \right) \frac{\partial T_s}{\partial x} dx dy \\ & + \left( \frac{U_{T,s}}{L_x L_y} \right) (T_w - T_s) dx dy + \left( \frac{U_{T,sw,s}}{L_x L_y} \right) (T_{sw,s} - T_s) dx dy \end{aligned} \quad (2)$$

where the upper sign is for parallel flow and the lower is for counterflow. The energy balance on the s fluid for multipass crossflow gives,

$$\begin{aligned} \left( \frac{W_s}{L_x L_y} c_{P,s} \right) \frac{\partial T_s}{\partial \theta} dx dy = & - \left( \frac{W_s}{L_x / N_p} c_{P,s} \right) \frac{\partial T_s}{\partial y} dx dy \\ & + \left( \frac{U_{T,s}}{L_x L_y} \right) (T_w - T_s) dx dy + \left( \frac{U_{T,sw,s}}{L_x L_y} \right) (T_{sw,s} - T_s) dx dy \end{aligned} \quad (3)$$



Three additional equations can be written for all flow arrangements from an energy balance on the separating wall and the two side walls:

$$\left(\frac{\bar{C}_w}{L_x L_y}\right) \frac{\partial T_w}{\partial \theta} dx dy = \left(\frac{U_{T,r}}{L_x L_y}\right) (T_r - T_w) dx dy + \left(\frac{U_{T,s}}{L_x L_y}\right) (T_s - T_w) dx dy \quad (4)$$

$$\left(\frac{\bar{C}_{sw,r}}{L_x L_y}\right) \frac{\partial T_{sw,r}}{\partial \theta} dx dy = \left(\frac{U_{T,sw,r}}{L_x L_y}\right) (T_r - T_{sw,r}) dx dy \quad (5)$$

$$\left(\frac{\bar{C}_{sw,s}}{L_x L_y}\right) \frac{\partial T_{sw,s}}{\partial \theta} dx dy = \left(\frac{U_{T,sw,s}}{L_x L_y}\right) (T_s - T_{sw,s}) dx dy \quad (6)$$

The preceding relations can be simplified to give the following set of five differential equations:

$$\left(W_r c_{p,r}\right) \frac{\partial T_r}{\partial \theta} = -\left(W_r c_{p,r}\right) \frac{\partial T_r}{\partial x} L_x + U_{T,r} (T_w - T_r) + U_{T,sw,r} (T_{sw,r} - T_r) \quad (7)$$

$$\left(W_s c_{p,s}\right) \frac{\partial T_s}{\partial \theta} = -W_s c_{p,s} \frac{\partial T_s}{\partial x} L_s + U_{T,s} (T_w - T_s) + U_{T,sw,s} (T_{sw,s} - T_s) \quad (8)$$

$$\bar{C}_w \frac{\partial T_w}{\partial \theta} = U_{T,r} (T_r - T_w) + U_{T,s} (T_s - T_w) \quad (9)$$

$$\bar{C}_{sw,r} \frac{\partial T_{sw,r}}{\partial \theta} = U_{T,sw,r} (T_r - T_{sw,r}) \quad (10)$$

$$\bar{C}_{sw,s} \frac{\partial T_{sw,s}}{\partial \theta} = U_{T,sw,s} (T_s - T_{sw,s}) \quad (11)$$

where for parallel flow,

$$\begin{aligned} \xi &\equiv x \\ L_s &\equiv L_x \end{aligned} \quad (12)$$

for counterflow

$$\begin{aligned} \xi &\equiv L_x - x \\ L_s &\equiv L_x \end{aligned} \quad (13)$$

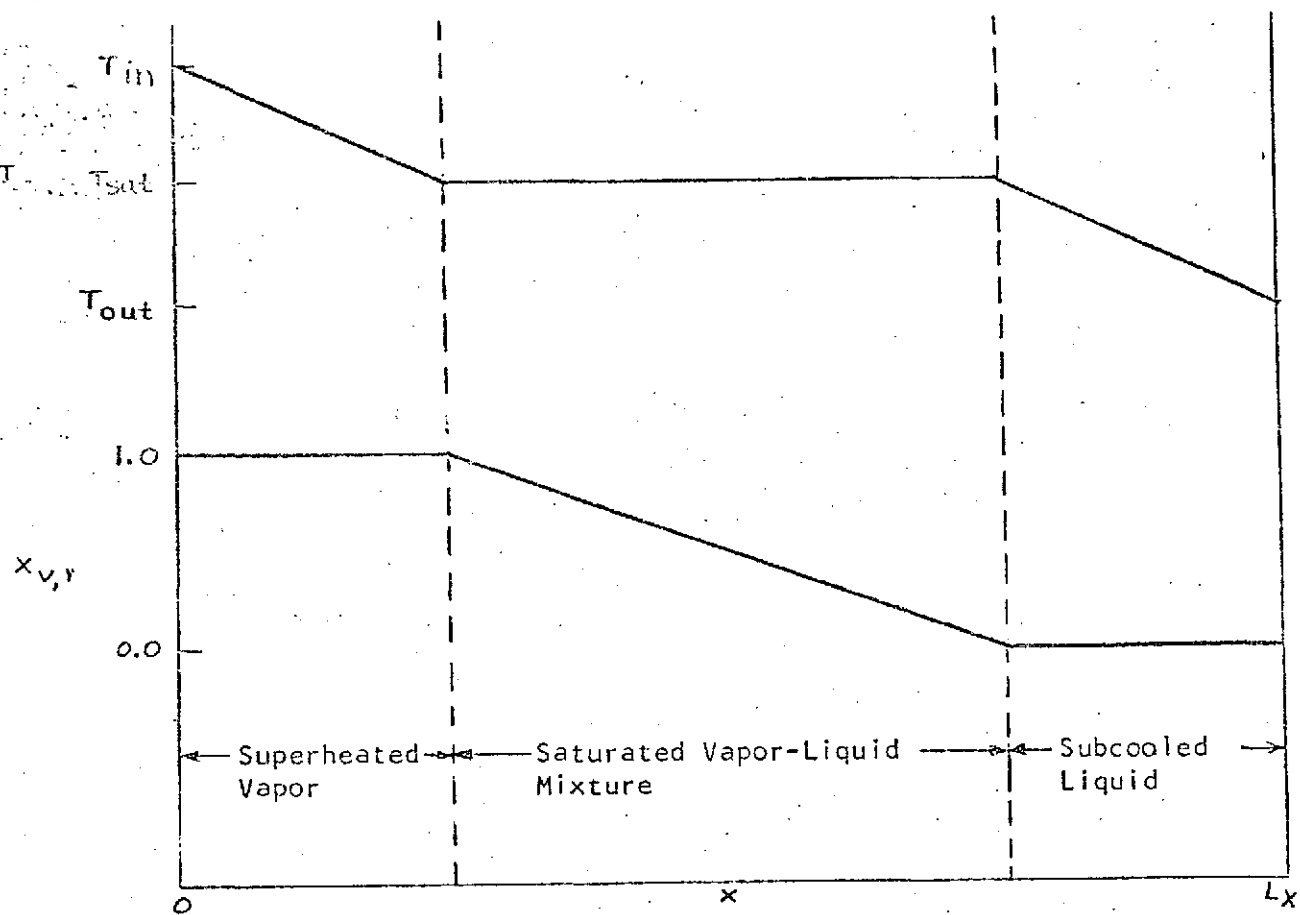
and for multipass crossflow,

$$\begin{aligned} \xi &\equiv y \\ L_s &\equiv N_p L_y \end{aligned} \quad (14)$$

This is the final system of differential equations which must be solved simultaneously to compute the transient performance of the heat exchanger.

#### Single-Component Condensing Fluid on r Side

For this case, an additional quantity, the vapor quality  $x_{v,r}$ , must be considered. The idealized temperature and quality distributions of the condensing r fluid at any time will be of the form shown in the sketch on the following page.



Idealized Temperature and Quality Distributions

In general, as can be seen from the above sketch, the heat exchanger involves three regions--two single-phase regions, where the  $r$  fluid is either a superheated vapor or a subcooled liquid, and one two-phase condensing region, where the  $r$  fluid is a mixture of saturated vapor and liquid. The equations which pertain and must be solved at any point depend on whether the region in which the point is located

is single-phase or two-phase.

The only equation which is affected by the fact that the  $r$  fluid is condensing is the one obtained from an energy balance on this fluid. This equation can be written in general terms as,

$$\begin{aligned} \left(\frac{W_r}{L_x L_y}\right) \frac{\partial H_r}{\partial \theta} dx dy = & - \left(\frac{W_r}{L_y}\right) \frac{\partial H_r}{\partial x} dx dy + \left(\frac{U_{T,r}}{L_x L_y}\right) (T_w - T_r) dx dy \\ & + \left(\frac{U_{T,sw,r}}{L_x L_y}\right) (T_{sw,r} - T_r) dx dy \end{aligned} \quad (15)$$

where  $H_r$  is the enthalpy per unit mass of the  $r$  fluid.

In each of the two single-phase regions, the vapor quality is constant and equal to either 1.0 or 0.0, depending on whether the fluid temperature is above (superheated vapor) or below (subcooled liquid) the saturation temperature,  $T_{sat}$ . The enthalpy of the  $r$  fluid in these regions can be expressed as,

$$H_r \equiv c_{p,r} T_r$$

where  $c_{p,r}$  is the specific heat corresponding to either vapor or liquid. As would be expected, Equation 15 reduces to the same expression as that obtained for the case of single-phase fluids on both sides (Equation 7). Therefore, the set of differential equations which must be solved consists of Equations 7 through 11.

In the two-phase region, the temperature of the  $r$  fluid is constant and equal to the saturation temperature. The enthalpy here can be expressed as

$$H_r \equiv h_{fg,sat} x_{v,r}$$

Substituting this definition of  $H_r$  into Equation 15 and simplifying the resulting expression, gives the differential equation governing the vapor-quality distribution:

$$\left( W_r h_{fg, sat} \right) \frac{\partial x_{v,r}}{\partial \theta} = - \left( w_r h_{fg, sat} \right) \frac{\partial x_{v,r}}{\partial x} L_x + U_{T,r} (T_w - T_r) + U_{T,sw,r} (T_{sw,r} - T_r) \quad (16)$$

The set of differential equations which must be solved in the two-phase region then consists of Equation 16 and Equations 8 through 11.

#### Wet Gas on r Side

For this case, the r fluid is considered to consist of two components--a noncondensing gas component (which can be a mixture of gases, for example, air) and a vapor component which partially condenses out of the stream in traversing the heat exchanger. Three of the equations obtained by consideration of the differential element are affected by the fact that the r fluid is a wet gas. These are the ones determined from energy balances on the r fluid, the separating wall, and the side wall in contact with the r fluid.

The derivations of the pertinent relations for this case are presented and discussed in Appendix III. The results of these derivations are the following differential equations:

$$\left( W_{g,r} C_{P,r}^* \right) \frac{\partial T_r}{\partial \theta} = - \left( w_{g,r} C_{P,r}^* \right) \frac{\partial T_r}{\partial x} L_x + U_{T,r} (T_w - T_r) + U_{T,sw,r} (T_{sw,r} - T_r) \quad (17)$$

$$\bar{C}_w \frac{\partial T_w}{\partial \theta} = U_{T,r} (T_r - T_w) + U_{T,s} (T_s - T_w)$$

$$- \bar{N}_{v,w} \left[ \left( h_{fg,DP} \frac{\partial w_r}{\partial T_r} \right) \left( W_{g,r} \frac{\partial T_r}{\partial \theta} + w_{g,r} \frac{\partial T_r}{\partial x} L_x \right) \right]$$

$$(18)$$

$$\bar{C}_{sw,r} \frac{\partial T_{sw,r}}{\partial \theta} = U_{T,sw,r} (T_r - T_{sw,r}) - (1 - \bar{N}_{v,w}) \left[ \left( h_{fg,DP} \frac{\partial \omega_r}{\partial T_r} \right) \left( w_{g,r} \frac{\partial T_r}{\partial \theta} + w_{g,r} \frac{\partial T_r}{\partial x} L_x \right) \right] \quad (19)$$

where

$$C_{P,r}^* = c_{p,g,r} + \omega_r c_{p,v,r} \quad (20)$$

and  $\bar{N}_{v,w}$ , defined as the fraction of the total condensation rate occurring on the separating wall, is approximated as,

$$\bar{N}_{v,w} = \frac{U_{T,r} |T_r - T_w|}{U_{T,r} |T_r - T_w| + U_{T,sw,r} |T_r - T_{sw,r}|} \quad (21)$$

By reason of the assumptions given on pages 6 through 7, the humidity and derivative of humidity with temperature appearing in the above expressions are known functions of only the fluid temperature. Therefore, the independent variables whose values are to be determined are the same as for the case of a single-phase  $r$  fluid; that is,  $T_r$ ,  $T_s$ ,  $T_w$ ,  $T_{sw,r}$ , and  $T_{sw,s}$ . The set of differential equations which must be solved for these variables then consists of Equations 17 through 19 and Equations 8 and 11.

#### Information Required for Solution of Equations

In order to solve the set of differential equations and compute the transient performance for any particular case, a variety of data must be specified. First of all, the physical boundaries

(dimensions) of the heat exchanger must be specified, along with the thermophysical properties of the two fluids and the geometric properties of the heat-transfer surfaces and exchanger structure. The remaining information required involves the specification of two sets of conditions. These come under the categories of initial and boundary conditions.

### Initial Conditions

These correspond to the distributions of fluid and metal temperature throughout the exchanger at the start of the transient ( $\theta = 0$ ). There is a choice of two initial temperature distributions corresponding to either a start-up or a steady-state situation.

For the case of start-up, the initial temperature of the metal throughout the exchanger is constant and equal to the temperature prevailing immediately prior to introducing the two fluids. The initial temperature distribution of each fluid is also constant and is taken as equal to the specified inlet temperature of the fluid at the start of the transient. The inlet temperatures of both fluids at  $\theta = 0$  will usually be specified to be equal to the given metal temperature; this is equivalent to assuming that the heat exchanger is initially filled with both fluids and is in thermal equilibrium.

Initial conditions corresponding to steady-state refer to a situation where the heat exchanger undergoes a transient change in operating conditions during steady-state operation. For this case, the initial steady-state temperature distributions are obtained by setting equal to zero all of the derivatives with respect to time in the differential equations. The resulting set of equations is then solved in the same manner as for the transient calculations. The distributions calculated in this fashion correspond to the steady-state conditions prevailing immediately prior to the initiation of the transient.

### Boundary Conditions

The information which must be supplied consists of the inlet conditions and flow rate of each fluid as functions of time. The fluid inlet conditions which must be specified consist of the temperature and, for a condensing fluid, vapor quality or absolute humidity. The inlet pressure must also be specified; however, this quantity is required to be constant with time. The prescribed variations must cover the time interval over which the transient performance is to be determined.



## SOLUTION PROCEDURE

The procedures developed for solving the governing differential equations and computing the transient thermal performance are described in this section. The manner in which the differential equations are rearranged to obtain a more convenient form is discussed first. The method used to solve for the temperature distributions at any given time,  $\theta$ , is then presented. Finally, the over-all procedure is described in general terms.

### Finite-Difference Representation of Equations

An examination of the governing differential equations shows that they are hyperbolic. Equations of this type can be solved by a finite-difference scheme either based on the method of characteristics (Ref 2) or based on taking incremental steps along the time scale ( $\theta$ ) and in the physical flow plane ( $X, Y$ ). A procedure based on the method of characteristics could be used to obtain accurate results for the temperature distributions at all values of time. However, with this method there is no freedom in choosing the finite-difference grid to be used in performing the calculations, since the relative spacing of the points is fixed by the characteristics of the particular problem being considered. As a result, this procedure is somewhat unwieldy and difficult to implement. The alternative approach of taking finite steps in  $\theta$ ,  $X$ , and  $Y$  is somewhat less accurate than the method of characteristics at small values of time; that is, values on the order of the dwell time of either of the two fluids. However, this approach is considerably more flexible than the method of characteristics and can be applied in a straightforward manner. In addition, by appropriately choosing the form in which the equations are written for purposes of solution, a procedure can be obtained which will provide acceptable accuracy in virtually all cases. For these reasons, a finite-difference procedure based on the time scale and the physical flow plane has been selected as being

the most practical for the application being considered.

The first step in obtaining the equations in a form suitable for numerical solution is to replace all of the derivatives with respect to time by their finite-difference approximations. The time derivative of temperature (or quality) in each equation is then written in the following form,

$$\frac{\partial T}{\partial \theta} = \frac{T - T'}{\theta - \theta'} = \left( \frac{1}{\Delta \theta} \right) (T - T') \quad (22)$$

where  $T'$  denotes the value at the beginning of the time step,  $\theta'$  and  $T$  denotes the value at the current time,  $\theta = \theta' + \Delta \theta$ .

The remainder of each equation can be written in terms of the values at the previous time ( $\theta'$ ) or the current time ( $\theta$ ).

For example, with the former approach (called the "explicit" method in this report) Equation 7 becomes,

$$\left( \frac{W_r c_{p,r}}{\Delta \theta} \right) (T_r - T'_r) = -w_r c_{p,r} \frac{\partial T'_r}{\partial x} L_x + U_{T,r} (T'_w - T'_r) + U_{T,sw,r} (T'_{sw,r} - T'_r) \quad (23)$$

and with the latter approach (called the "implicit" method) it becomes

$$\left( \frac{W_r c_{p,r}}{\Delta \theta} \right) (T_r - T'_r) = -w_r c_{p,r} \frac{\partial T_r}{\partial x} L_x + U_{T,r} (T_w - T_r) + U_{T,sw,r} (T_{sw,r} - T_r) \quad (24)$$

These are termed the explicit and implicit methods, respectively, since they correspond roughly to the methods of the same name used in expressing the governing equations for the analogous case of transient conduction. (Ref 3).

The implicit method has been selected as the more practical approach, since it allows considerable latitude in choosing the size

of the time step required to achieve acceptable accuracy. However, in order to obtain sufficiently accurate solutions of the transient performance, it is still advisable to bound the value of the time increment used in carrying out the calculations. This is done by computing approximate minimum and maximum acceptable values of  $\Delta\theta$  ( $\Delta\theta_{min}$  and  $\Delta\theta_{max}$ , respectively) and insuring that the actual value used for the calculations at any time  $\theta$  is between these limits. The suggested procedures for determining the values of  $\Delta\theta_{min}$  and  $\Delta\theta_{max}$  are presented and discussed in Appendix IV. In actual fact, it is not strictly necessary to adhere rigidly to these limits. Experience has shown that any reasonable value of  $\Delta\theta$  will yield sufficiently accurate results, particularly in comparison to the accuracy of the empirical data available for the calculation of the heat-transfer coefficients.

#### Reduced Equations

As was described previously the calculation of the transient performance for any particular case involves the simultaneous solution of a set of five differential equations. In order to simplify the solution procedure it is convenient to reduce the number of equations to two, one related to each of the two fluids. This is accomplished by replacing each derivative with respect to time by its finite-difference approximation and then algebraically eliminating the values of the metal temperatures at the current time ( $T_w$ ,  $T_{sw,r}$ , and  $T_{sw,s}$ ) from the equations by substitution.

The rather lengthy algebraic procedure involved in reducing the system of equations is described in Appendix V. The equations which are obtained as a result of these manipulations are as follows. The expression related to the s fluid is,

$$\frac{\partial T_s}{\partial \theta} + P_s T_s = Q_{s1} + Q_{s2} T_r$$

(25)

The corresponding relation for the r fluid is,

$$\frac{\partial T_r}{\partial x} + P_r T_r = Q_{r1} + Q_{r2} T_s \quad (26)$$

For cases where the r fluid is single phase or a wet gas. If the r fluid is a single-component condensing fluid, Equation 26 does not apply in the two-phase region, since there the temperature is known and equal to the saturation temperature ( $T_{sat}$ ). In the two-phase region the vapor quality replaces the temperature of the r fluid as the unknown quantity, and the following expression is used instead of Equation 26:

$$\frac{\partial x_{v,r}}{\partial x} + P_{TP} x_{v,r} = Q_{TP1} + Q_{TP2} T_s \quad (27)$$

The coefficients in the above expression ( $P_s$ ,  $Q_{s1}$ ,  $Q_{s2}$ ,  $P_r$ ,  $Q_{r1}$ ,  $Q_{r2}$ ,  $P_{TP}$ ,  $Q_{TP1}$ , and  $Q_{TP2}$ ) consist of combinations of fluid properties and flow rates, heat-transfer conductances, geometric properties, metal capacitances, and the value of the time increment. In addition the coefficients,  $Q_{s1}$ ,  $Q_{r1}$ , and  $Q_{TP1}$  contain the known values of the metal and fluid temperatures (and quality in the two-phase region) at the beginning of the time step,  $\theta'$ . Expressions for all of these coefficients are derived and presented in Appendix V.

The determination of the fluid temperature (and quality) distributions at any time  $\theta$  now involves the simultaneous solution of a pair of differential equations, Equations 25 and 26 or 25 and 27. This is accomplished by integrating consecutively the relation for each fluid with respect to distance along its flow length from the known conditions at its inlet to the outlet. The iterative solution

procedure used is described subsequently in this section. Once the fluid temperature (and quality) distributions at time  $\theta$  have been determined, the corresponding metal temperature distributions are computed with the following reduced equations,

$$T_W = F_{W1} * T_r + F_{W2} T_s + F_{W3} T'_W + F_{W,\theta,L} T_r' - F_{W,x,L} \frac{\partial T_r}{\partial x} \quad (28)$$

$$T_{SW,r} = F_{SWr1} * T_r + F_{SWr2} T'_{SW,r} + F_{SW,\theta,L} T_r' - F_{SW,x,L} \frac{\partial T_r}{\partial x} \quad (29)$$

$$T_{SW,s} = F_{SWs1} T_s + F_{SWs2} T'_{SW,s} \quad (30)$$

These expressions and the relations for the coefficients are also presented in Appendix V.

#### Integration of Reduced Differential Equations

There are a variety of schemes available for numerically integrating the differential equation for each of the two fluids. The procedure chosen is semi-analytical in nature. In performing the integration for each fluid, the procedure requires estimating the temperature distribution of the fluid on the other side, and making some estimate of the spatial variation (between calculation stations equally spaced along the flow length) of this temperature and the coefficients.

#### Calculation Sections

For the purpose of carrying out the numerical integration of the differential equations, the heat exchanger is considered to be divided into a number of calculation sections. The manner in which this division is performed for parallel-flow and counterflow configurations is illustrated in Figure 1. In these configurations

all properties are assumed to be constant in the transverse (  $Y$  ) direction. In the following descriptions the subscript, "  $i$  ", refers to properties either at station  $i$  or in section  $i$  (see Fig 1). For example,  $T_{r,i}$  is the temperature of the  $r$  fluid at station  $i$  ;  $\bar{P}_{r,i}$  is the average value of the coefficient,  $P_r$  , in section  $i$  . For a multipass-crossflow configuration, the division of the exchanger is as illustrated in Figure 2 for the specific case of a three-pass arrangement. In each section all properties are assumed to be constant in the direction transverse to the flow direction; that is, along the  $\Delta y$  face of the section for the  $r$  fluid and along the  $\Delta x$  face for the  $s$  fluid. The subscripts, "  $i$  " and "  $j$  " refer to either stations or sections. For example, the temperature  $T_{r,i,j}$  is the temperature of the  $r$  fluid at station  $i$  along the  $\Delta y$  face of section  $i,j$  ; similarly, for the  $s$  fluid,  $T_{s,i,j}$  is the temperature at station  $j$  along the  $\Delta x$  face of section  $i,j$  . Also, as was the case for parallel flow and counterflow,  $\bar{P}_{r,i,j}$  is the average value of the coefficient  $P_r$  in section  $i,j$  .

The sizes of the spatial increments,  $\Delta x$  and  $\Delta y$  , used affect the accuracy of the solutions obtained. Experience has shown, however, that the use of a reasonable number of sections (say,  $N_x=10$  for parallel flow or counterflow and  $N_{x,p}=5, N_{y,p}=10$  for crossflow) yields results which are sufficiently accurate for most applications.

### Integrated Equations

The approach involved in obtaining the integrated forms of the reduced differential equations is illustrated here by considering the relation for the  $r$  fluid, Equation 26, for a parallel-flow or counterflow configuration. In integrating this expression over the general section  $i$  (that is, from station  $i$  to  $i+1$  along the flow length), or  $x_i$  to  $x_{i+1}$  it is assumed that the coefficients  $P_r$  and  $Q_{r2}$  are constant and equal to the average values,

$$\bar{P}_{r,i} = \frac{1}{2} (P_{r,i} + P_{r,i+1})$$

(31)

$$\bar{Q}_{r2,i} = \frac{1}{2} (Q_{r2,i} + Q_{r2,i+1})$$

(32)

In addition, it is assumed that  $Q_{r1}$  and  $T_s$  vary linearly with distance,

$$Q_{r1} = a_Q + b_Q x$$

$$T_s = a_T + b_T x$$

where the constants,  $a_Q$ ,  $b_Q$ ,  $a_T$ , and  $b_T$ , are computed from the values of  $Q_{r1}$  and  $T_s$  at stations  $i$  and  $i+1$ .

With these assumptions, Equation 26 in the section  $i$  takes the form,

$$\frac{\partial T_r}{\partial x} + \bar{P}_{r,i} T_r = \alpha_i + \beta_i x$$

(33)

where the constants  $\alpha_i$  and  $\beta_i$  are defined as,

$$\alpha_i \equiv a_Q + \bar{Q}_{r2,i} a_T$$

(34)

$$\beta_i \equiv b_Q + \bar{Q}_{r2,i} b_T$$

(35)

Equation 33 is now seen to be a linear differential equation of the first order and first degree for which the integrating factor is  $e^{\bar{P}_{r,i} x}$  (pp 40-43 of Ref 4). Applying this factor, integrating between  $X_i$  and  $X_{i+1} = X_i + \Delta x$ , and substituting for the various constants yields the final integrated form of Equation 26,

$$T_{r,i+1} = (T_{r,i}) e^{-\bar{P}_{r,i} \Delta x} + \frac{1}{\bar{P}_{r,i}} \left\{ \left[ Q_{r1,i+1} + \bar{Q}_{r2,i} T_{s,i+1} \right] \left[ 1 - \frac{1}{\bar{P}_{r,i} \Delta x} \left( 1 - e^{-\bar{P}_{r,i} \Delta x} \right) \right] \right. \\ \left. + \left[ Q_{r1,i} + \bar{Q}_{r2,i} T_{s,i} \right] \left[ \frac{1}{\bar{P}_{r,i} \Delta x} \left( 1 - e^{-\bar{P}_{r,i} \Delta x} \right) - e^{-\bar{P}_{r,i} \Delta x} \right] \right\} \quad (36)$$

With calculated values of the coefficients and estimated values of  $T_s$  at all stations, this expression can be used to compute the temperature distribution of the  $r$  fluid starting with the known condition at the inlet ( $T_r = T_{in,r}$  at  $i = 1$ ) and proceeding to the outlet ( $i+1 = N_x + 1$ ).

The same procedure as described above is used to obtain the final integrated forms of Equations 25 and 27:

$$T_{s,k+1} = (T_{s,k}) e^{-\bar{P}_{s,k} \Delta x} + \frac{1}{\bar{P}_{s,k}} \left\{ \left[ Q_{s1,k+1} + \bar{Q}_{s2,k} T_{r,k+1} \right] \left[ 1 - \frac{1}{\bar{P}_{s,k} \Delta x} \left( 1 - e^{-\bar{P}_{s,k} \Delta x} \right) \right] \right. \\ \left. + \left[ Q_{s1,k} + \bar{Q}_{s2,k} T_{r,k} \right] \left[ \frac{1}{\bar{P}_{s,k} \Delta x} \left( 1 - e^{-\bar{P}_{s,k} \Delta x} \right) - e^{-\bar{P}_{s,k} \Delta x} \right] \right\} \quad (37)$$

where  $k \equiv i$  for parallel flow and  $k \equiv N_x + 2 - i$  for counterflow and

$$x_{v,r,i+1} = (x_{v,r,i}) e^{-\bar{P}_{TP,i} \Delta x} + \frac{1}{\bar{P}_{TP,i}} \left\{ \left[ Q_{TP1,i+1} + \bar{Q}_{TP2,i} T_{s,i+1} \right] \left[ 1 - \frac{1}{\bar{P}_{TP,i} \Delta x} \left( 1 - e^{-\bar{P}_{TP,i} \Delta x} \right) \right] \right. \\ \left. + \left[ Q_{TP1,i} + \bar{Q}_{TP2,i} T_{s,i} \right] \left[ \frac{1}{\bar{P}_{TP,i} \Delta x} \left( 1 - e^{-\bar{P}_{TP,i} \Delta x} \right) - e^{-\bar{P}_{TP,i} \Delta x} \right] \right\} \quad (38)$$



Expressions similar to Equations 36 through 38 are obtained for the case of a crossflow configuration, the only difference being in the method of subscripting.

### Iterative Solution Procedure

As was mentioned previously, an iterative solution of the equations is required to determine the temperature (and quality) distributions at any time  $\theta$  from the known distributions at the beginning of the time step ( $\theta' = \theta - \Delta\theta$ ). The procedure selected for performing this iterative solution is called the "double-sweep" method. With this method, each iteration involves two integrations (or calculation sweeps), one in each of the two general flow directions of the exchanger. The major steps of the general procedure are described briefly below for the specific case of a counterflow configuration in which both fluids are single-phase. The steps in the procedures for other flow arrangements are essentially the same and can be inferred easily from the following description:

1. The temperature distributions of both fluids are estimated. For the calculations at any time  $\theta$ , the initial estimates are taken as the distributions computed for the previous time  $\theta'$ . Subsequent estimates are based on the results of each iteration.
2. The coefficients,  $P_r$ ,  $Q_{r1}$ ,  $Q_{r2}$ , and  $P_s$ ,  $Q_{s1}$ ,  $Q_{s2}$ , are evaluated at each calculation station using the estimated values of temperature.
3. Keeping the estimated temperature distribution of the s fluid fixed, the temperature distribution of the r fluid is calculated. This calculation is performed using Equation 36 to determine the values of  $T_r$  at each station along the flow length, starting with the specified temperature at the r fluid inlet ( $T_{in,r}$ ) for time  $\theta$ .
4. Keeping the temperature distribution of the r fluid fixed at the values computed in Step 3, the temperature distribution

of the s fluid is calculated. This calculation is performed using Equation 37 to determine the values of  $T_s$  at each station, starting with the specified temperature at the s fluid inlet ( $T_{in,s}$ ) for time  $\theta$ .

5. Steps 1 through 4 are repeated until the distributions calculated in Steps 3 and 4 are within a specified tolerance of the corresponding distributions estimated in Step 1. When this condition has been satisfied and converged fluid temperature distributions have been obtained, the coefficients in Equations 28 through 30 are evaluated at each calculation station. These coefficients and expressions are then used to compute the metal temperatures,  $T_w$ ,  $T_{sw,r}$ , and  $T_{sw,s}$ , at each station.

Essentially the same procedure as described above is used for cases where the r fluid is condensing. One difference is that the integration over the section in which condensation starts is performed in two parts. For the portion of the section upstream of the point of condensation (that is, from the adjacent upstream station to the condensation point) the coefficients are taken to be constant and equal to their values at the adjacent upstream calculation station. Similarly, for the downstream portion of the section, the coefficients are taken to be equal to their values at the adjacent downstream station. This is done because the values of the coefficients in the non-condensing region differ significantly from those in the condensing region (due to the different heat-transfer mechanisms prevailing), requiring that the condensation point be treated as a discontinuity. Another necessary variation in the procedure is due to the fact that the coefficients are relatively strong functions of the temperature (and quality) distributions of the condensing fluid. Therefore, in the iteration procedure, two consecutive calculation sweeps are made keeping the s fluid temperature distribution fixed instead of one as in the case of two single-phase fluids. This is done so as to "update" the values of the coefficients in each iteration before computing the temperatures of the s fluid. In this manner, a more stable approach to the solution is obtained.

### General Description of Over-All Procedure

The stepwise procedure used to determine the transient performance over a specified time interval is based on the methods previously described and is quite straightforward in nature. First, the temperature distributions at the start of the transient (initial conditions at time zero) are determined. The magnitude of the first time increment,  $\Delta \theta$ , is then chosen and used to define the new time,  $\theta$ . The inlet conditions and flow rates (boundary conditions) of both fluids are evaluated at  $\theta$  from the prescribed variations of these quantities. The double-sweep method of iteration described above is then used to determine the new distributions. The pressure drops of both fluids are calculated iteratively, along with the temperature distributions, using the methods and relations described on pages 98-99 and 117-120 of Reference 5. This procedure, starting with the selection of the time increment and determining the transient performance of the exchanger at the new value of  $\theta$ , is repeated until the desired time interval has been covered.

## CONCLUSIONS AND RECOMMENDATIONS

### Conclusions

Analytical models have been developed which represent single-phase or condensing heat-transfer processes in compact plate-fin heat exchangers operating under transient conditions. The differential equations governing the transient thermal performance have been derived on the basis of these models for parallel-flow, counterflow, and multipass-crossflow configurations. A finite-difference procedure, which involves using the implicit or backward-difference method of representing the derivatives with respect to time, has been developed for the simultaneous solution of the governing equations. This procedure can be used to obtain the temperature distributions in the exchanger at any time  $\theta$  for given initial conditions and prescribed variations with time of the inlet conditions and flow rates of both streams. Although the procedure is iterative in nature, it has proved quite effective in obtaining sufficiently stable and accurate solutions. Finally, this has led to the development of a general method of analyzing the transient behavior of the types of compact heat exchanger most often encountered in aerospace applications.

### Recommendations

Due to the hyperbolic nature of the transient differential equations (see Appendix IV), any finite-difference solution procedure based on the physical flow plane may not yield accurate results for the temperature distributions at values of time less than the dwell time of either of the two fluids. However, solutions for the outlet conditions at all times greater than say twice the dwell time are sufficiently accurate for all practical applications. In most cases, the response time of each fluid is one or two orders of magnitude larger than its dwell time. If the solution procedure is being used as an analytical tool to investigate the over-all dynamic behavior of the heat exchanger, the main points of interest are the general shapes

of the transient performance curves and the values of the response times. The minor inaccuracies inherent at small values of time will then be negligible and will have no significant effect on the results obtained. However, there may be applications where it is desirable to compute the detailed temperature distributions throughout the exchanger at all values of time; for example, in performing a stress analysis of a given heat-exchanger configuration to determine whether the transient thermal stresses are within acceptable limits. If such applications are of interest, it is recommended that a new procedure be developed for the solution of the transient differential equations which would satisfy this requirement. This would involve using the method of characteristics (Ref 2) to determine the detailed temperature distributions at all values of time within the interval of interest.

In treating heat exchangers in which condensation from a wet gas occurs, the procedure for determining the transient performance involves assuming that the heat-transfer coefficient can be evaluated using experimental data obtained with a single-phase fluid. This assumption is expected to be quite reasonable for the vast majority of cases involving this type of exchanger. However, there is very little known about the mechanism of condensation from a wet gas and its effects on the heat-transfer process. Since this situation occurs in many aerospace applications, it seems important that basic information on its effects be obtained. Therefore, it is recommended that an analytical and experimental program be initiated to provide a better understanding of the phenomenon of condensation from a wet gas.

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FIGURES

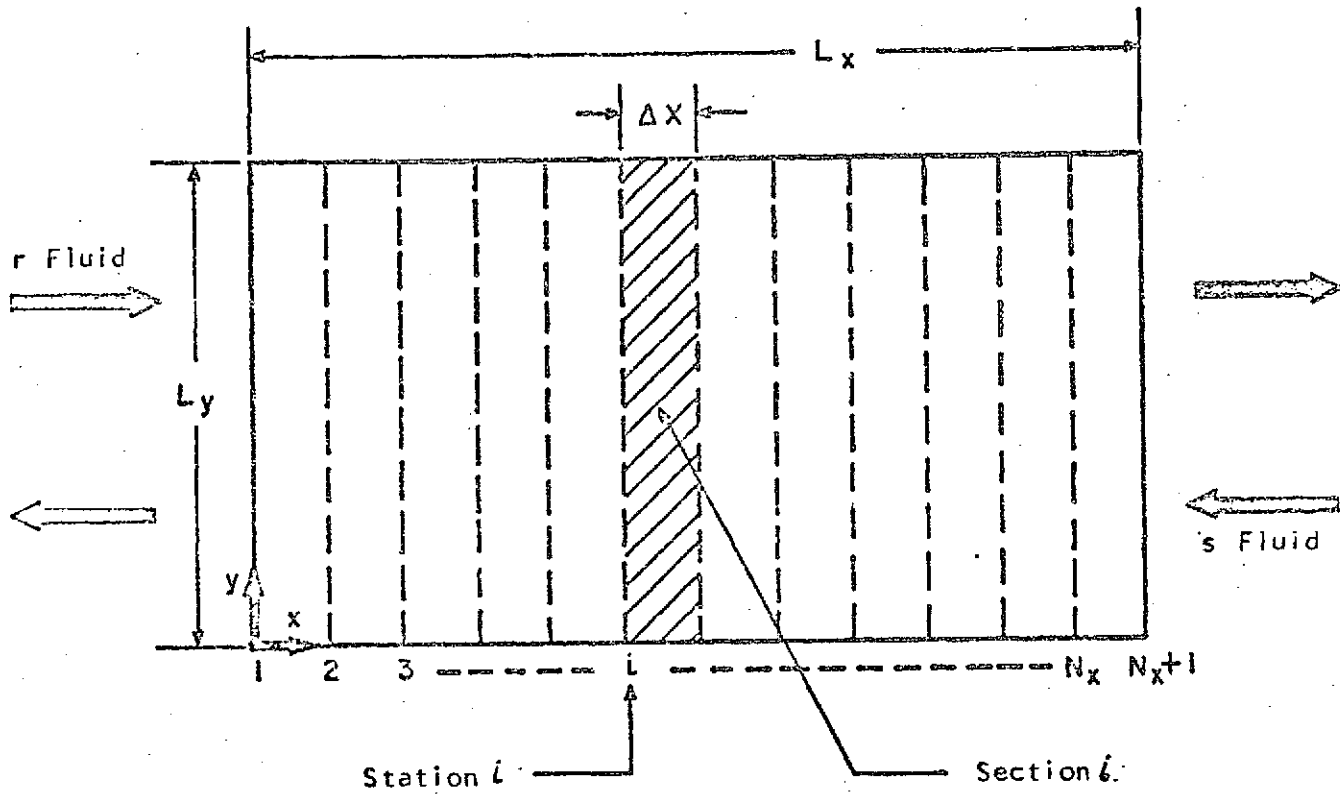


FIGURE 1 - REPRESENTATION OF FLOW FIELD  
FOR PARALLEL FLOW AND COUNTERFLOW



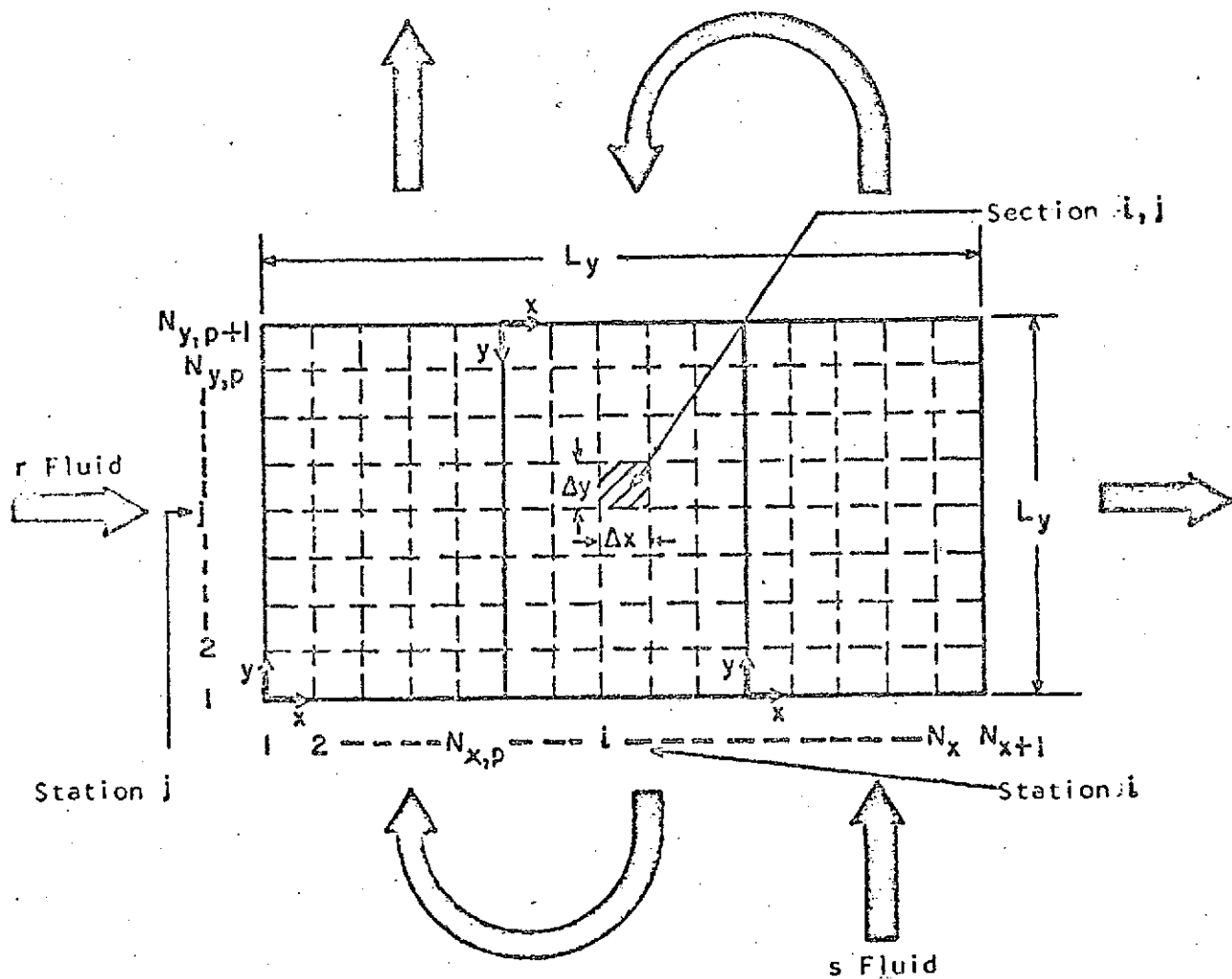


FIGURE 2 - REPRESENTATION OF FLOW FIELD  
FOR MULTIPASS CROSSFLOW

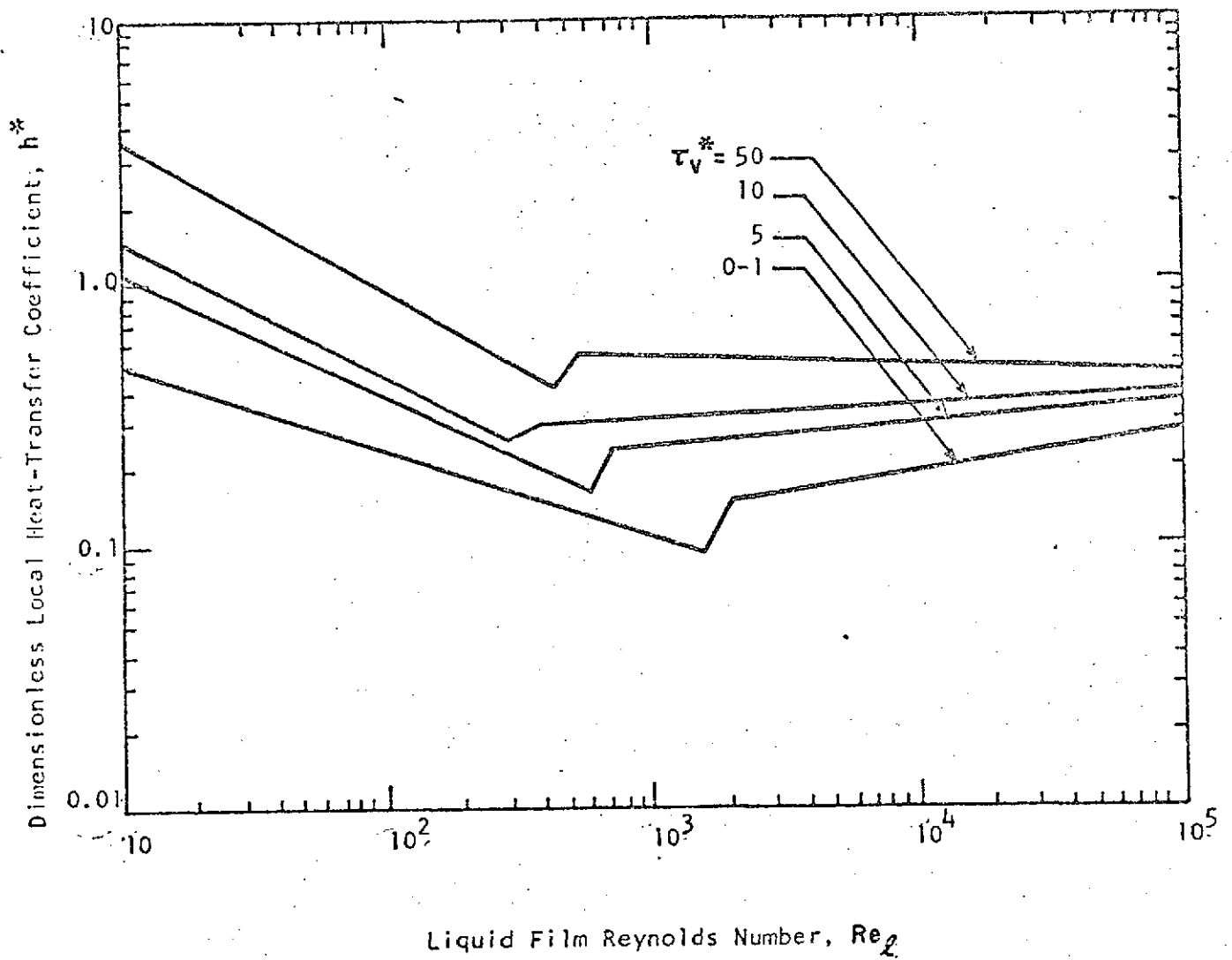


FIGURE 3 - APPROXIMATE EFFECT OF VAPOR SHEAR ON LOCAL  
CONDENSING HEAT-TRANSFER COEFFICIENT FOR  $\rho_{r2}=1$

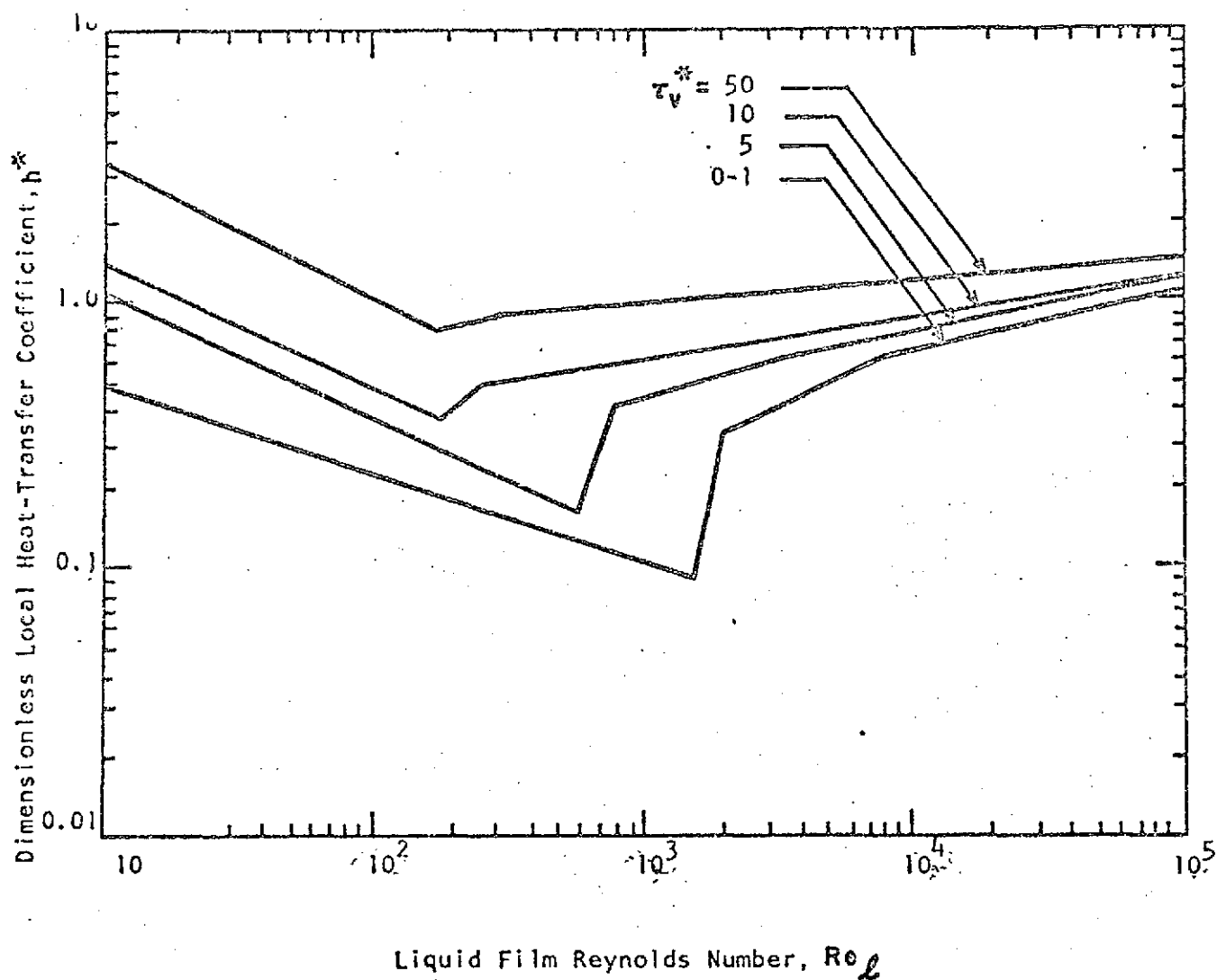


FIGURE 4 - APPROXIMATE EFFECT OF VAPOR SHEAR ON LOCAL  
 CONDENSING HEAT-TRANSFER COEFFICIENT FOR  $Pr_l = 10$

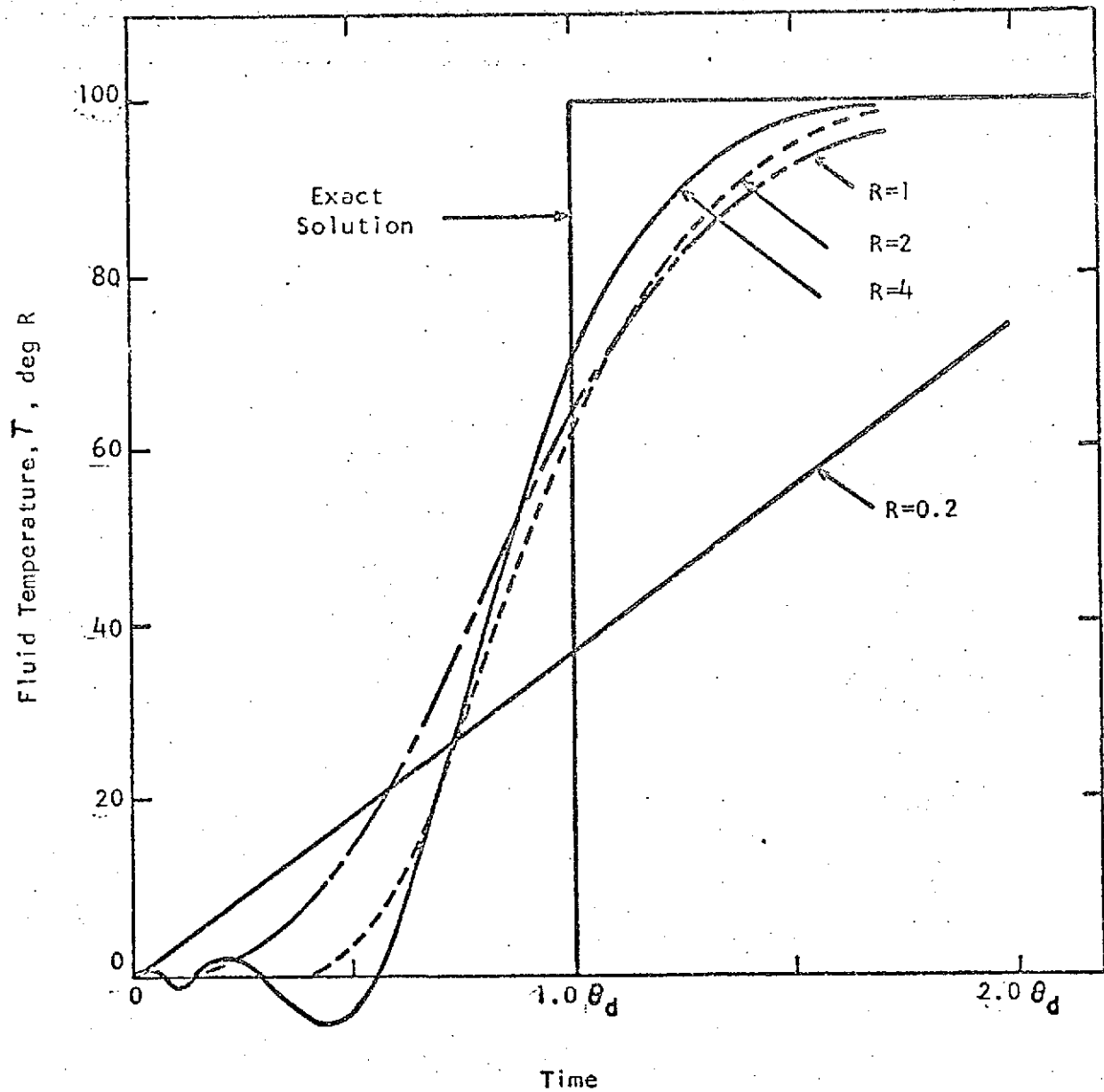


FIGURE 5 - COMPARISON OF FINITE-DIFFERENCE AND EXACT SOLUTIONS FOR AN ADIABATIC DUCT WITH A STEP CHANGE IN INLET FLUID TEMPERATURE

NOMENCLATURE

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
$A$	Heat transfer area	sq ft
$A_c$	Minimum free flow area	sq ft
$A_f/A$	Ratio of fin to total heat-transfer area	- -
$A_H$	Heat-transfer area per unit area in the flow plane for a matrix	- -
$A_{sw,l}$	Heat-transfer area of side wall parallel to parting plates	sq ft
$A_{sw,t}$	Heat-transfer area of side wall normal to parting plates	sq ft
$a$	Parting-plate thickness	ft
$a_{sp}$	Splitter-plate thickness	ft
$a_{sw,l}$	Plate thickness of side wall parallel to parting plates	ft
$a_{sw,t}$	Plate thickness of side wall normal to parting plates	ft
$b$	Plate spacing (distance between adjacent plates) for a matrix	ft
$\bar{C}_w$	Total heat capacity of exchanger core structure	Btu per deg R
$\bar{C}_{sw}$	Heat capacity of side wall in contact with one fluid	Btu per deg R
$c_p$	Specific heat	Btu per lbm deg R
$f$	Fanning friction factor	- -
$G$	Mass velocity ( $\equiv w/A_c$ )	lbm per hr sq ft
$g$	Acceleration due to gravity	ft per hr <sup>2</sup>
$g_c$	Proportionality constant in Newton's Law ( $4.169 \times 10^8$ )	ft lbm per lbf hr <sup>2</sup>

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
H	Enthalpy per unit mass	Btu per lbm
h	Heat transfer coefficient	Btu per hr sq ft deg R
$h_{fg}$	Latent heat (heat of vaporization)	Btu per lbm
j	Colburn modulus	- -
k	Thermal conductivity	Btu per hr ft deg R
L	Flow length or total heat- exchanger core dimension	ft
$L_f$	Effective fin length	ft
M	Molecular weight	lbm per lbm-mole
$N_L$	Number of layers or sandwiches for a matrix	- -
$N_P$	Number of passes in a multipass crossflow exchanger	- -
$N_V$	Rate of condensation for a wet gas	lbm per hr
$\bar{N}_{V,W}$	Fraction of condensation from a wet gas occurring on separating walls and attached fins	- -
$N_X$	Total number of length increments into which the x dimension ( $L_X$ ) of an exchanger is divided	- -
$N_{Y,P}$	Total number of length increments into which the y dimension per pass ( $L_Y$ ) of a crossflow exchanger is divided	- -
$Pr$	Prandtl number ( $\equiv c_p \mu / k$ )	- -
P	Pressure	lbf per sq ft
R	Time-increment parameter defined in Appendix IV	- -

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
$Re$	Reynolds number ( $\equiv 4r_h G/\mu$ )	- -
$r_h$	Hydraulic radius ( $\equiv A_c L/A$ )	ft
$S_f$	Fin spacing	ft
$T$	Temperature	deg R
$U_T$	Heat-transfer conductance on one side	Btu per hr deg R
$V$	Volume	cu ft
$(V_f/V)_{int}$	Ratio of metal volume to total volume between plates for a matrix	- -
$V_{void}$	Void volume on one side	cu ft
$W$	Fluid inventory on one side	lbm
$w$	Mass flow rate	lbm per hr
$x$	Coordinate axis of exchanger	ft
$x_v$	Vapor quality (mass-fraction vapor) of a single-component condensing fluid	- -
$y$	Coordinate axis of exchanger	ft
$\Delta p$	Pressure drop	lbf per sq ft
$\Delta x$	Length increment in $x$ direction	ft
$\Delta y$	Length increment in $y$ direction	ft
$\Delta \theta$	Time increment	hr
$\delta$	Effective fin thickness	ft
$\eta_F$	Fin efficiency	- -
$\eta_o$	Heat-transfer surface efficiency	- -
$\theta$	Time	hr
$\theta'$	Time at beginning of time step	hr

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
$\theta_d$	Dwell time of one fluid in heat exchanger ( $\equiv W/W$ )	hr
$\mu$	Dynamic viscosity	lbm per hr ft
$\xi$	Distance along flow direction of s fluid	ft
$\rho$	Density	lbm per cu ft
$\sigma'$	Ratio of minimum free-flow area to total face area for one side	- -
$\sigma_{int}$	Ratio of minimum free-flow area to face area between adjacent plates	- -
$\tau_v$	Vapor shear stress	lbf per sq ft
$\omega$	Absolute humidity of a wet gas (lbm moisture/lbm dry gas)	- -

<u>Subscript</u>	<u>Description</u>
DP	Value at dew point of wet gas
g	Gas component of wet gas
i	Station or section in the X direction
in	Inlet
j	Station or section in the Y direction for a crossflow configuration
L	Liquid
m	Mean
max	Maximum
met	Metal
min	Minimum
NF	Nonflow dimension

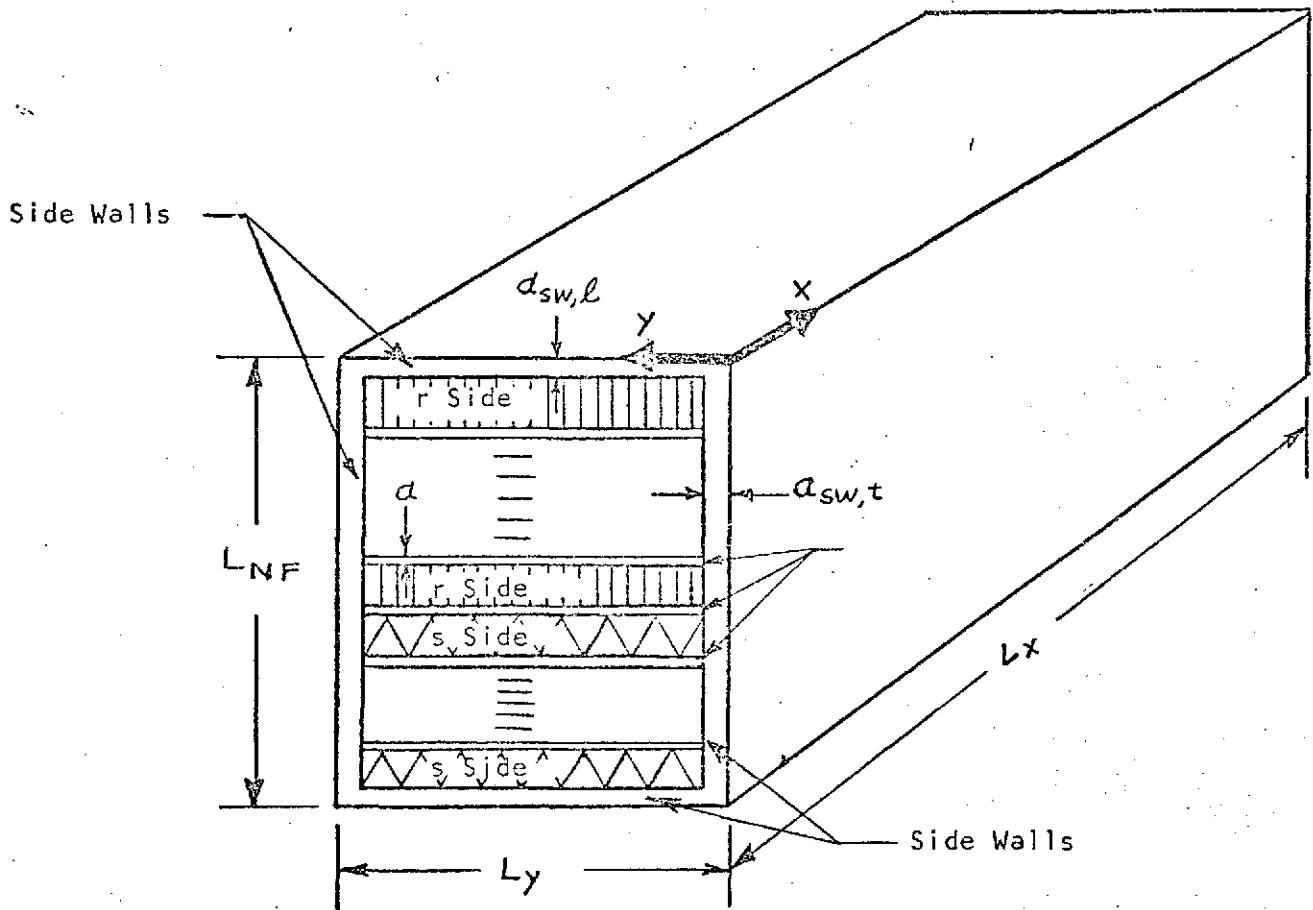


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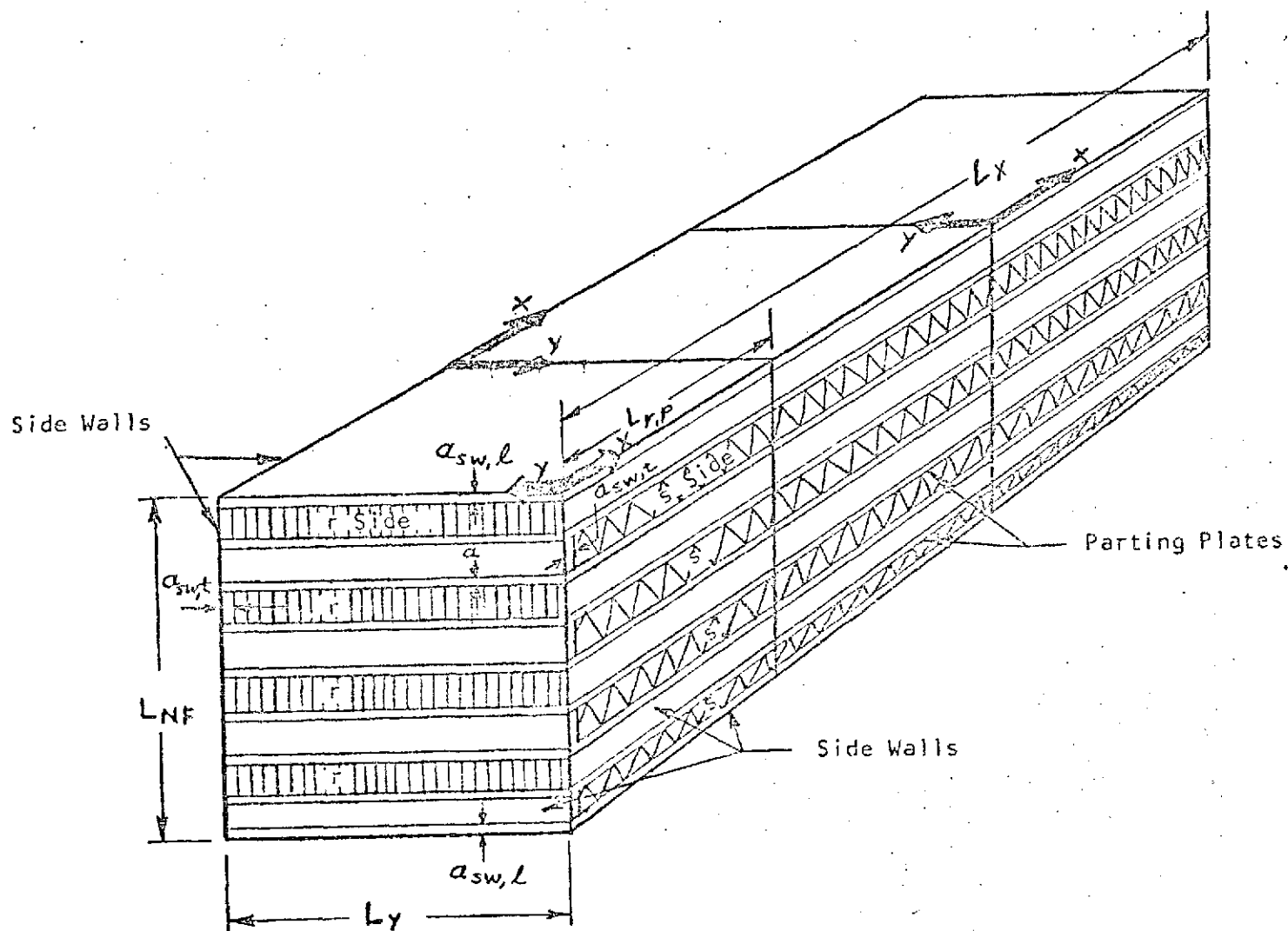
out	Outlet
r	One side or fluid of heat exchanger
s	Other side or fluid of heat exchanger
sat	Value at saturated conditions
sw	Side Wall
v	Vapor
w	Separating Wall
x	Coordinate axis of exchanger
y	Coordinate axis of exchanger

SuperscriptDescription

o	Quantity nondimensionalized by hydraulic radius
1	Value at beginning of time step ( $\theta' = \theta - \Delta \theta$ )



Parallel Flow and Counterflow



Multipass Crossflow

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APPENDICES

## APPENDIX I

### CALCULATION OF GEOMETRIC PROPERTIES

Expressions are presented in this appendix for computing the major geometric properties of the heat exchanger which are required for the calculation of the transient performance. Several geometric quantities related to the heat-transfer matrix (surface) associated with each fluid appear in these expressions; for example, the ratios of the total heat-transfer area of one passage to the surface area of one plate ( $A_H$ ), the minimum free-flow area to face area between adjacent plates ( $\sigma_{int}$ ), and the metal volume to total volume between adjacent plates ( $(V_f/V)_{int}$ ). The relations used to compute these and other pertinent geometric variables are derived and presented in Appendix I of Reference 5, for the different types of plate-fin matrices commonly used.

#### Void Volumes

The void volume on each side ( $V_{void}$ ) is used in computing the inventory of each fluid. The expression for computing the void volume on the  $r$  side is,

$$V_{void,r} = \sigma_r' V \quad (1-1)$$

where  $V$  is the total volume,

$$V = L_x L_y L_{NF} \quad (1-2)$$

The ratio of minimum free-flow area to total face area for the  $r$  side,  $\sigma_r'$ , is computed as,

$$\sigma_r' = \frac{\sigma_{int,r} b_r N_{L,r}}{b_T} \quad (1-3)$$

where,

$$b_T \equiv b_r N_{L,r} + (N_{L,r} - 1) a_{sp,r} + b_s N_{L,s} + (N_{L,s} - 1) a_{sp,s} + 2a \quad (1-4)$$

The corresponding relations for the s side are the same as those on the previous page with the "r" subscript replaced by "s".

#### Heat-Transfer Areas

The total heat-transfer area in contact with each fluid involves two categories--the surface area associated with the side walls and the surface area associated with the separating wall and attached fins.

#### Side Walls

The heat-transfer area of the side walls in contact with each fluid is calculated in two parts corresponding to the area associated with the side wall parallel to the parting plates ( $A_{sw,\ell}$ ) and normal to the parting plates ( $A_{sw,t}$ ). The area  $A_{sw,\ell}$  simply consists of half the total surface area of one passage (including the fins); the expression for the r side is

$$A_{sw,\ell,r} = \frac{1}{2} A_{H,r} L_x L_y \quad (1-5)$$

The relation for  $A_{sw,t}$  on the r side is,

$$A_{sw,t,r} = B_{sw} b_r N_{L,r} \quad (1-6)$$

where

$$B_{sw} \equiv \frac{2L_x L_{NF}}{b_T} \quad (1-7)$$

The corresponding expressions for the s side are the same except that the "r" subscript is replaced by "s".

#### Separating Wall and Fins

The heat-transfer area on each side is obtained from the definition of the hydraulic radius. For the r side, the expression is,

$$A_r = \frac{A_{c,r} L_x}{r_{h,r}} - \frac{1}{2} A_{H,r} L_x L_y \quad (1-8)$$

where the total minimum free-flow area,  $A_{c,r}$ , is computed as,

$$A_{c,r} = \sigma'_r L_{NF} L_y \quad (1-9)$$

Note that in calculating this quantity, the area associated with the side wall parallel to the parting plates is subtracted from the nominal heat-transfer area for the fluid.

The expression used to compute the heat-transfer area on the s side depends on the flow arrangement. For parallel-flow and counterflow,

$$A_s = \frac{A_{c,s} L_x}{r_{h,s}} - \frac{1}{2} A_{H,s} L_x L_y \quad (1-10)$$

where

$$A_{c,s} = \sigma'_s L_{NF} L_y \quad (1-11)$$

For multipass crossflow,

$$A_s = \frac{A_{c,s} N_P L_y}{r_{h,s}} - \frac{1}{2} A_{H,s} L_x L_y \quad (1-12)$$

where

$$A_{c,s} = \sigma_s' L_{NF} \left( \frac{L_x}{N_P} \right) \quad (1-13)$$

#### Metal Capacitances

The capacitance of the separating walls and attached fins is given by the expression.

$$\bar{C}_w = \left[ \left( \rho_{met} c_{P,met} V_{rat} \right)_r + \left( \rho_{met} c_{P,met} V_{rat} \right)_s + \rho_{met,P} c_{P,met,P} V_{rat,P} \right] V \quad (1-14)$$

where

$$V_{rat,r} = \frac{1}{b_T} \left[ \left( \frac{V_f}{V} \right)_{int,r} b_r N_{L,r} + (N_{L,r} - 1) a_{SP,r} \right] \quad (1-15)$$

$$V_{rat,s} = \frac{1}{b_T} \left[ \left( \frac{V_f}{V} \right)_{int,s} b_s N_{L,s} + (N_{L,s} - 1) a_{SP,s} \right] \quad (1-16)$$

$$V_{rat,P} = \frac{2a}{b_T} \quad (1-17)$$



The relation for the capacitance of the side walls in contact with the  $r$  fluid is,

$$\bar{C}_{sw,r} = L_x L_y \left\{ (\rho_{met} c_{p,met})_{sw} a_{sw,l} + \frac{1}{2} (\rho_{met} c_{p,met})_r \left[ \left( \frac{V_f}{V} \right)_{int,r} b_r N_{L,r} + (N_{L,r} - 1) a_{sp,r} \right] \right\} \\ + (\rho_{met} c_{p,met})_{sw} a_{sw,t} B_{sw} \left[ b_r N_{L,r} + (N_{L,r} - 1) a_{sp,r} + a \right] \quad (1-18)$$

The corresponding expression for  $C_{sw,s}$  is the same as Equation 1-18 with the "r" subscript replaced by "s".

## APPENDIX II

### CALCULATION OF TOTAL CONDUCTANCES

This appendix presents the relations and procedures used to determine the values of the total conductances for heat transfer between each fluid and the metal surfaces with which it is in contact. First the expressions defining the conductances for a side are defined. This is followed by a description of the procedures for calculating the major terms in these expressions, the heat-transfer coefficient (  $h$  ) and fin efficiency (  $\eta_o$  ).

#### Definition

For heat transfer between either fluid and the separating wall (including the fins) the conductance is defined as

$$U_T \equiv \eta_o h A \quad (11-1)$$

For the side walls in contact with the fluid, the conductance is given by the relation,

$$U_{T,sw} \equiv \eta_o h A_{sw,l} + h A_{sw,t} \quad (11-2)$$

In the above expressions, the term  $\eta_o$  is the over-all surface efficiency defined as,

$$\eta_o = 1 - (A_f/A)(1 - \eta_f) \quad (11-3)$$

#### Heat-Transfer Coefficient

##### Single-Phase Fluid or Wet Gas

The procedure for computing the value of  $\eta$  used in

Equations 11-1 and 11-2 is the same whether the fluid is single-phase or a condensing wet gas. In evaluating the heat-transfer coefficient, it is assumed that a table of experimental values of friction factor (  $f$  ) and Colburn modulus (  $j$  ) versus Reynolds number (  $Re$  ) is available for the heat-transfer matrix (surface) through which the fluid flows. In using this table, the local Reynolds number,

$$Re = \frac{4r_h G}{\mu} \quad (11-4)$$

is computed. A mean value of the Colburn modulus,  $j_m$ , is then obtained by interpolation from the table assuming,

$$j_m = \frac{H_c}{Re^{a_c}} \quad (11-5)$$

where  $H_c$  and  $a_c$  are constants of interpolation obtained from the tabulated values of  $Re$  and  $j$ . The value of  $j_m$  is corrected for the effect of temperature-dependent fluid properties to determine the final value of  $j$ . For a gas,

$$j = j_m \left( \frac{T_w}{T} \right)^n \quad (11-6)$$

where  $T_w$  is the local wall temperature. Similarly, for a liquid,

$$j = j_m \left( \frac{\mu_w}{\mu} \right)^n \quad (11-7)$$

where  $\mu_w$  is the viscosity evaluated at  $T_w$ . The exponent,  $n$ ,

is a function of the flow regime and whether the fluid is being heated or cooled (Ref 5, pp 90-91). When the final corrected value of  $j$  has been obtained, the heat-transfer coefficient is calculated as;

$$h = \frac{j G c_p}{Pr^{2/3}} \quad (11-8)$$

#### Single-Component Condensing Fluid

The procedure used to calculate  $h$  for this case depends on the local state of the fluid. In the superheated-vapor region (see sketch on page 13), the procedure is the same as that described above for a single-phase fluid. In the two-phase region, the procedure is based on the correlations of Rohsenow, Webber and Ling (Ref 6) and is essentially the same as that described on pages 115-117 of Reference 4 and pages 179-180 of Reference 7, the only exception being that local rather than mean values of  $h$  are obtained; this procedure, with  $x_v = 0$ , is also used to compute  $h$  in the subcooled-liquid region.

The method of Rohsenow et al. yields values of the mean heat-transfer coefficient,  $h_m$ , over the distance between the point where condensation starts to the point being considered,  $Z_L$ . Curves obtained using this method are presented on Figure 103 of Reference 7. These show the variation of the nondimensional mean heat-transfer coefficient,  $h_m^*$ , versus the nondimensional condensing length,  $Z_L^*$ , and liquid Reynolds number,  $Re_L$ , with nondimensional shear stress  $\tau_v^*$  as a parameter for two values of liquid Prandtl number,  $Pr_L = 1$  and  $Pr_L = 10$ . In the finite-difference calculations described in this report, it is more convenient to be able to evaluate the local condensing heat-transfer coefficient,  $h$ . In order to do this, the plots of  $h_m^*$  were graphically differentiated to obtain the corresponding plots of  $h^*$ . The resulting curves of

$h^*$  versus  $Re_L$  are approximately linear on a log-log plot and are shown on Figures 3 and 4. The procedure for determining  $h$  then involves calculating the local values of  $Pr_L$ ,  $\tau_v^*$ , and  $Re_L$  as

$$Pr_L = \frac{c_{p,L} \mu_L}{k_L} \quad (11-9)$$

$$\tau_v^* = \frac{\left[ \frac{g}{(\mu_L / \rho_L)^2} \right]^{1/3}}{g(\rho_L - \rho_v)} \tau_v \quad (11-10)$$

$$Re_L = (1 - X_v) \frac{4 r_h G}{\mu_L} \quad (11-11)$$

where the vapor shear stress ( $\tau_v$ ) is evaluated by the procedures described on pages 115-117 of Reference 5. These quantities are then used to interpolate among the various curves and obtain the value of  $h^*$ . Finally, the local heat-transfer coefficient is calculated as,

$$h = k_L \left[ \frac{g}{(\mu_L / \rho_L)^2} \right]^{1/3} h^* \quad (11-12)$$

### Fin Efficiency

#### Single-Phase or Single-Component Condensing Fluid

For a heat-transfer matrix arranged in a single-sandwich configuration, the fin efficiency is calculated as,

$$\eta_f = \frac{\tanh h(m l_f)}{(m l_f)} \quad (11-13)$$

where  $m$  is the fin-efficiency parameter defined as,

$$m \equiv \sqrt{\frac{zh}{k_{met} \delta}} \quad (11-14)$$

The expressions used to compute  $\eta_f$  for double and triple sandwich arrangements are given in Appendix II of Reference 1.

#### Wet Gas

The relations used to compute  $\eta_f$  for the case of a wet gas are exactly the same as those mentioned above except that the fin-efficiency parameter is calculated as,

$$m \equiv \sqrt{\frac{zh^*}{k_{met} \delta}} \quad (11-15)$$

The quantity,  $h^*$ , is an "effective" heat-transfer coefficient defined so as to approximately account for the effect of condensation on the temperature distributions in the fins. The expression for  $h^*$  is derived below.

For the case of a condensing wet gas, the heat-transfer rate (  $q$  ) to the general surface area,  $A$ , can be written as,

$$q = hA(T - T_w) + h_{fg} N_v$$

where  $N_v$  is the rate of condensation (see Appendix III) (11-16)

given by the relation,

$$N_V = - \left( W_g \frac{\partial \omega}{\partial T} \right) \frac{\partial T}{\partial \theta} - \left( W_g \frac{\partial \omega}{\partial T} \right) \frac{\partial T}{\partial x} L_x \quad (11-17)$$

Substituting Equation 11-17 into Equation 11-16 and using the finite-difference approximation of the time derivative gives,

$$\dot{Q} = \left\{ h - \frac{h_{fg} \left( \frac{\partial \omega}{\partial T} \right)}{A(T - T_w)} \left[ \frac{W_g}{\Delta \theta} (T - T') + W_g L_x \frac{\partial T}{\partial x} \right] \right\} A (T - T_w) \quad (11-18)$$

Comparison of Equation 11-18 with the corresponding relation for the single-phase case,

$$\dot{Q} = h^* A (T - T_w) \quad (11-19)$$

shows that the effective heat-transfer coefficient is given by the expression,

$$h^* \equiv h + \frac{h_{fg} \left( \frac{\partial \omega}{\partial T} \right)}{A(T - T_w)} \left[ - \frac{W_g}{\Delta \theta} (T - T') - W_g L_x \frac{\partial T}{\partial x} \right] \quad (11-20)$$

### APPENDIX III

#### DERIVATION OF TRANSIENT EQUATIONS FOR A WET GAS

In this appendix, the transient differential equations for the case of a condensing wet gas are derived and presented. Expressions for computing the humidity ( $\omega$ ) and derivative of humidity with respect to fluid temperature ( $\partial\omega/\partial T$ ) are also given.

#### Fluid Energy Balance

Considering a general differential element, the energy balance on the condensing wet gas (r fluid) gives,

$$\begin{aligned} \frac{\partial (W_g H_g + W_v H_v)}{\partial \theta} \left( \frac{dx dy}{L_x L_y} \right) = & - \frac{\partial (w_g H_g + w_v H_v)}{\partial x} \left( \frac{dx dy}{L_y} \right) + \left( \frac{U_{T,r}}{L_x L_y} \right) (T_w - T_r) dx dy \\ & + \left( \frac{U_{T,sw,r}}{L_x L_y} \right) (T_{sw,r} - T_r) dx dy - H_v N_v \left( \frac{dx dy}{L_x L_y} \right) \end{aligned} \quad (III-1)$$

The enthalpies of the gas and vapor components,  $H_g$  and  $H_v$ , are

$$H_g = C_{P,g} (T_r - T_o) \quad (III-2)$$

$$H_v = C_{P,v} (T_r - T_{LFP}) + h_{fg,DP} + C_{P,L} (T_{DP} - T_o) \quad (III-3)$$



where  $T_0$  is some reference temperature at which both enthalpies are defined as being zero. The expression for the condensation rate,  $N_v$ , is obtained from a mass balance as,

$$N_v = - \frac{\partial W_v}{\partial \theta} - \frac{\partial W_v}{\partial x} L_x \quad (III-4)$$

Now, from the definition of absolute humidity,

$$W_v = \omega W_g$$

$$w_v = \omega w_g \quad (III-5)$$

Assuming  $W_g$  is approximately constant (that is, the local value of gas density does not change significantly) we can write

$$\frac{\partial (W_g H_g + W_v H_v)}{\partial \theta} = W_g \frac{\partial (H_g + \omega H_v)}{\partial \theta} = \frac{\partial H_g}{\partial \theta} + \omega \frac{\partial H_v}{\partial \theta} + H_v \frac{\partial \omega}{\partial \theta} \quad (III-6)$$

$$\frac{\partial (w_g H_g + w_v H_v)}{\partial x} = w_g \frac{\partial (H_g + \omega H_v)}{\partial x} = \frac{\partial H_g}{\partial x} + \omega \frac{\partial H_v}{\partial x} + H_v \frac{\partial \omega}{\partial x} \quad (III-7)$$

$$N_v = -W_g \frac{\partial \omega}{\partial \theta} - w_g \frac{\partial \omega}{\partial x} L_x \quad (III-8)$$

Substituting Equations III-6 through III-8 into Equation III-1 and simplifying the resulting expression gives,

$$W_g \left[ \frac{\partial H_g}{\partial \theta} + \omega \frac{\partial H_v}{\partial \theta} \right] = -w_g \left[ \frac{\partial H_g}{\partial x} + \omega \frac{\partial H_v}{\partial x} \right] L_x$$

$$+ U_{T,r} (T_w - T_r) + U_{T,sw,r} (T_{sw,r} - T_r) \quad (III-9)$$

The expressions for  $H_g$  and  $H_v$  can be differentiated to obtain,

$$\begin{aligned}\frac{\partial H_g}{\partial \theta} &= c_{p,g} \frac{\partial T_r}{\partial \theta} & \frac{\partial H_g}{\partial x} &= c_{p,g} \frac{\partial T_r}{\partial x} \\ \frac{\partial H_v}{\partial \theta} &= c_{p,v} \frac{\partial T_r}{\partial \theta} & \frac{\partial H_v}{\partial x} &= c_{p,v} \frac{\partial T_r}{\partial x}\end{aligned}$$

(III-10)

Then, if we define,

$$c_{p,r}^* \equiv c_{p,g} + \omega c_{p,v}$$

(III-11)

the final form of Equation III-9 is obtained as,

$$\left( W_g c_{p,r}^* \right) \frac{\partial T_r}{\partial \theta} = - \left( W_g c_{p,r}^* \right) \frac{\partial T_r}{\partial x} L_x + U_{T,r} (T_w - T_r) + U_{T,sw,r} (T_{sw,r} - T_r)$$

(III-12)

#### Wall Energy Balance

Energy balances on the separating wall and side walls in contact with the condensing wet gas give the following two expressions,

$$\begin{aligned}\left( \frac{\bar{C}_w}{L_x L_y} \right) \frac{\partial T_w}{\partial \theta} dx dy &= \left( \frac{U_{T,r}}{L_x L_y} \right) (T_r - T_w) + \left( \frac{U_{T,s}}{L_x L_y} \right) (T_s - T_w) \\ &+ (H_v - H_{e,w}) N_{v,w} \left( \frac{dx dy}{L_x L_y} \right) - \left( \frac{W_{e,w}}{L_y} \right) \frac{\partial H_{e,w}}{\partial x} dx dy\end{aligned}$$

(III-13)

$$\begin{aligned}\left( \frac{\bar{C}_{sw,r}}{L_x L_y} \right) \frac{\partial T_{sw,r}}{\partial \theta} dx dy &= \left( \frac{U_{T,sw,r}}{L_x L_y} \right) (T_r - T_{sw,r}) + (H_v - H_{e,sw}) N_{v,sw} \left( \frac{dx dy}{L_x L_y} \right) \\ &- \left( \frac{W_{e,sw}}{L_y} \right) \frac{\partial H_{e,sw}}{\partial x} dx dy\end{aligned}$$

(III-14)

In these relations, the terms  $N_{V,W}$  and  $N_{V,SW}$  are the condensation rates on the separating and side walls, respectively,

$$N_{V,W} + N_{V,SW} = N_V$$

(III-15)

In addition, the terms  $W_{L,W}$  and  $W_{L,SW}$  represent the liquid flow rates along the walls. Using the definitions of enthalpy in Equations III-13 and III-14 and simplifying the resulting expressions gives,

$$\bar{C}_W \frac{\partial T_W}{\partial \theta} = U_{T,r}(T_r - T_W) + U_{T,s}(T_s - T_W) + \left[ C_{P,V}(T_r - T_{DP}) + h_{fg,DP} + C_{P,L}(T_{DP} - T_W) \right] N_{V,W} - (W_{L,W} C_{P,L}) \frac{\partial T_W}{\partial x} L_x$$

(III-16)

$$\bar{C}_{SW,r} \frac{\partial T_{SW,r}}{\partial \theta} = U_{T,SW,r}(T_r - T_{SW,r}) + \left[ C_{P,V}(T_r - T_{DP}) + h_{fg,DP} + C_{P,L}(T_{DP} - T_{SW,r}) \right] N_{V,SW} - (W_{L,SW} C_{P,L}) \frac{\partial T_{SW,r}}{\partial x} L_x$$

(III-17)

A number of reasonable simplifying assumptions can be made to reduce the complexity of the above relations. First of all, it is reasonable to assume that there is not a large variation with distance in either of the two wall temperatures. In addition, it is likely that the liquid flow rates along the walls will be small. Therefore, the terms  $(W_{L,W} C_{P,L}) \frac{\partial T_W}{\partial x} L_x$  and  $(W_{L,SW} C_{P,L}) \frac{\partial T_{SW,r}}{\partial x} L_x$  should be negligible compared to the other terms in the equations and can be ignored with little loss in accuracy. An additional assumption which is quite reasonable is that the terms  $C_{P,V}(T_r - T_{DP}) + C_{P,L}(T_{DP} - T_W)$  and  $C_{P,V}(T_r - T_{DP}) + C_{P,L}(T_{DP} - T_{SW,r})$  are each much smaller than the latent heat of vaporization,  $h_{fg,DP}$ . With these assumptions, Equations III-16 and III-17 reduce to

$$\bar{C}_w \frac{\partial T_w}{\partial \theta} = U_{T,r} (T_r - T_w) + U_{T,s} (T_s - T_w) + h_{fg,DP} N_{v,w}$$

(III-18)

$$\bar{C}_{sw,r} \frac{\partial T_{sw,r}}{\partial \theta} = U_{T,sw,r} (T_r - T_{sw,r}) + h_{fg,DP} N_{v,sw}$$

(III-19)

If we define the fraction of the total condensation rate occurring on the separating wall as  $\bar{N}_{v,w}$  then,

$$N_{v,w} = \bar{N}_{v,w} N_v$$

(III-20)

$$N_{v,sw} = (1 - \bar{N}_{v,w}) N_v$$

(III-21)

An accurate approximation of  $\bar{N}_{v,w}$  can be obtained by assuming that  $N_{v,w} \propto U_T (T_r - T_w)$  and  $N_{v,sw} \propto U_{T,sw,r} (T_r - T_{sw,r})$ . The term  $\bar{N}_{v,w}$  is then given by the relation,

$$\bar{N}_{v,w} = \frac{U_{T,r} (T_r - T_w)}{U_{T,r} (T_r - T_w) + U_{T,sw,r} (T_r - T_{sw,r})}$$

(III-22)

Now with the assumptions listed on page 7 of the main text, the humidity is a function of fluid temperature only. Therefore, the derivatives of humidity in the expression for the total condensation rate (Equation III-8) can be written as,

$$\begin{aligned} \frac{\partial \omega}{\partial \theta} &= \frac{\partial \omega_r}{\partial T_r} \cdot \frac{\partial T_r}{\partial \theta} \\ \frac{\partial \omega}{\partial x} &= \frac{\partial \omega_r}{\partial T_r} \cdot \frac{\partial T_r}{\partial x} \end{aligned}$$

(III-23)

The relation for  $N_v$  then becomes

$$N_v = - \left( \frac{\partial \omega}{\partial T} \right) \left[ W_g \frac{\partial T_r}{\partial \theta} + w_g \frac{\partial T_r}{\partial x} L_x \right] \quad (III-24)$$

The above expressions are then used in Equations III-18 and III-19 to obtain the final forms of the transient differential equations for the walls:

$$\begin{aligned} \bar{C}_w \frac{\partial T_w}{\partial \theta} = & U_{T,r}(T_r - T_w) + U_{T,s}(T_s - T_w) \\ & - \bar{N}_{v,w} \left[ \left( h_{fg,DP} \frac{\partial \omega_r}{\partial T_r} \right) \left( W_g \frac{\partial T_r}{\partial \theta} + w_g \frac{\partial T_r}{\partial x} L_x \right) \right] \end{aligned} \quad (III-25)$$

$$\bar{C}_{sw,r} \frac{\partial T_{sw,r}}{\partial \theta} = U_{T,sw,r}(T_r - T_{sw,r}) - (1 - \bar{N}_{v,w}) \left[ \left( h_{fg,DP} \frac{\partial \omega_r}{\partial T_r} \right) \left( W_g \frac{\partial T_r}{\partial \theta} + w_g \frac{\partial T_r}{\partial x} L_x \right) \right] \quad (III-26)$$

#### Evaluation of $\omega$ and $\partial \omega / \partial T$

The absolute humidity of any mixture can be expressed in terms of the partial pressures of the two components as,

$$\omega = \frac{p_p M_v}{(p - p_p) M_g} \quad (III-27)$$

where  $p_p$  is the partial pressure of the vapor component,  $p$  is the local pressure of the mixture, and  $M_v$  and  $M_g$  are the molecular weights of the vapor and gas components. At all temperatures below the dew point, the gas is assumed to be saturated. For a saturated mixture,

$$p_p = p_v$$

(III-28)

where  $p_v$  is the vapor pressure of the vapor component evaluated at the local fluid temperature. Equation III-27 then becomes,

$$\omega = \frac{p_v \left( \frac{M_v}{M_g} \right)}{p - p_v} \quad (\text{III-29})$$

For most substances, the vapor-pressure variation with temperature can be expressed quite accurately over a moderate temperature range as,

$$\ln(p_v) = \ln a_p - \frac{b_p}{T} \quad (\text{III-30})$$

or

$$p_v = a_p e^{-\frac{b_p}{T}} \quad (\text{III-31})$$

where  $a_p$  and  $b_p$  are constants which can be evaluated locally from tabulated values of  $p_v$  versus  $T$  for the vapor component.

Substituting Equation III-31 into III-27 gives the following expression for  $\omega$  as a function of fluid temperature,

$$\omega = \frac{\left( \frac{M_v}{M_g} \right)}{\left[ \left( \frac{p}{a_p} \right) e^{(b_p/T)} - 1 \right]} \quad (\text{III-32})$$

This is then differentiated with respect to  $T$  to obtain the final relation for  $\partial\omega/\partial T$ ,

$$\frac{\partial\omega}{\partial T} = \left[ \frac{b_p}{\left( \frac{M_v}{M_g} \right) T^2} \right] \left[ \omega \left( \frac{M_v}{M_g} + \omega \right) \right] \quad (\text{III-33})$$

For values of fluid temperature above the dew point, the humidity is constant ( $\partial\omega/\partial T = 0$ ) and is equal to the inlet value. Below the dew point,  $\omega$  and  $\partial\omega/\partial T$  are computed by Equations III-29 and III-33, respectively, using tabulated values of  $p_v$  versus  $T$  to obtain  $p_v$  and  $b_p$ .

## APPENDIX IV

### FACTORS AFFECTING CHOICE OF TIME INCREMENT

The size of the time increment used in the finite-difference solution of the transient equations at any time  $\theta$  affects to some extent the accuracy obtained. Experience has shown that the effect of  $\Delta\theta$  is moderate and considerable latitude exists in the choice of the value used. However, in order to achieve reasonable accuracy, it is desirable to bound  $\Delta\theta$ ; that is, to insure that the value selected is between calculated minimum and maximum values. The suggested procedures for computing  $\Delta\theta_{\min}$  and  $\Delta\theta_{\max}$  are presented in this appendix.

#### Calculation of Minimum Time Increment

Intuitively, one would expect the accuracy of the finite-difference solutions to improve as the size of the time increment is decreased; that is, one would not expect there to be a limiting minimum value of  $\Delta\theta$ . However, examination of the transient equations involved shows that they are hyperbolic in nature. Therefore, results obtained using any finite-difference procedure based on taking incremental steps along the time scale and in the physical flow plane are likely to be somewhat inaccurate for small values of time; that is, values of  $\theta$  on the order of the dwell times of the fluids in the heat exchanger.

This aspect of the transient equations can be illustrated by considering the case of flow through a constant wall temperature duct. The governing equation for this case is,

$$\frac{1}{v} \frac{\partial T}{\partial \theta} = -\frac{\partial T}{\partial z} - \alpha T$$

(IV-1)

where  $v$  is the fluid velocity,  $z$  is the distance along the flow length (  $L$  ), and



$$T \equiv T_{fluid} - T_{wall}$$

$$\alpha \equiv \frac{h(A/L)}{w c_p}$$

(IV-2)

This equation has a single characteristic given by

$$z_{char} = v \theta_{char}$$

(IV-3)

The characteristics represent lines in the  $z-\theta$  plane along which discontinuities in the derivatives of  $T$  may exist.

The finite-difference approximations in the selected solution procedure are roughly equivalent to

$$\frac{1}{v} \frac{T - T'}{\Delta \theta} = - \frac{\partial T}{\partial z} - \alpha T$$

$$\frac{\partial T}{\partial z} = - \left( \frac{1}{v \Delta \theta} + \alpha \right) T + \left( \frac{1}{v \Delta \theta} \right) T'$$

$$\frac{T_{i+1} - T_i}{\Delta z} = - \left( \frac{1}{v \Delta \theta} + \alpha \right) \left( \frac{T_i + T_{i+1}}{2} \right) + \left( \frac{1}{v \Delta \theta} \right) \left( \frac{T'_i + T'_{i+1}}{2} \right)$$

giving a recurrence formula of

$$T_{i+1} = \frac{1 - \frac{R + \alpha \Delta z}{2}}{1 + \frac{R + \alpha \Delta z}{2}} T_i + \frac{R}{2 + R + \alpha \Delta z} (T'_i + T'_{i+1})$$

(IV-4)

where the subscript "i" refers to equally spaced calculation stations along the flow length and the term,  $R$ , is defined as,

$$R \equiv \frac{\Delta z}{v \Delta \theta}$$

(IV-5)

Equation IV-5 can be written in terms of the fluid dwell time ( $\theta_d = W/w$ ) by recognizing that  $v = L/\theta_d$ . The resulting expression is then,

$$R \equiv \frac{(\Delta z/L)}{\Delta \theta} \theta_d$$

(IV-6)

The question is how to select the time increment in order to achieve optimum accuracy in the numerical solution. We will consider the simple case where  $\alpha = 0$  and choose five equal spatial increments in the duct. The selected initial and boundary conditions are

$$\begin{aligned} T(0, z) &= 0 \\ T(\theta, 0) &= 100 \end{aligned}$$

In this case, the exact solution for the outlet temperature ( $T_5$ ) is,

$$T_{OUT} = 0 \quad \text{for } \theta < \theta_d$$

$$T_{OUT} = 100 \quad \text{for } \theta \geq \theta_d$$

We will compare this with the finite-difference solutions for four values of  $R$  : 0.2, 1, 2 and 4. The appropriate recurrence formulas are obtained from Equation IV-4 as,

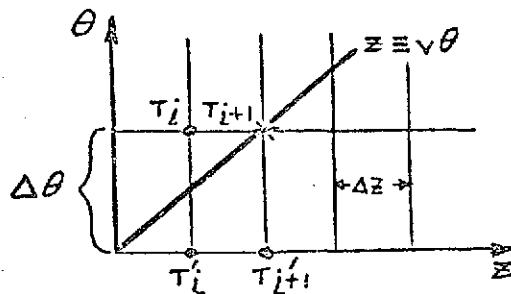
$$T_{i+1} = 0.81 T_i + 0.091 (T_{i+1}' + T_i') \quad \text{for } R = 0.2$$

$$T_{i+1} = \frac{1}{3} T_i + \frac{1}{3} (T_{i+1}' + T_i') \quad \text{for } R = 1$$

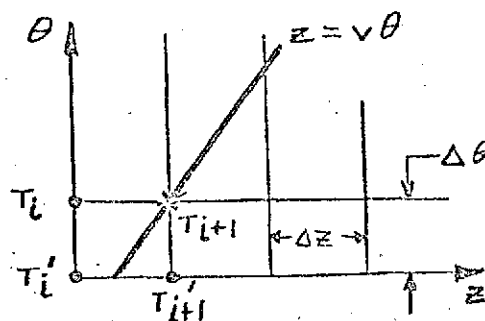
$$T_{i+1} = \frac{1}{2} (T_{i+1}' + T_i') \quad \text{for } R = 2$$

$$T_{i+1} = -\frac{1}{3} T_i + \frac{2}{3} (T_{i+1}' + T_i') \quad \text{for } R = 4$$

The calculated outlet temperatures are shown plotted in Figure 5 from which it can be observed that  $R = 1, 2$ , or  $4$  give a much better approximation to the exact solution than  $R = 0.2$ . The reason for this lies in the relationship between the finite-difference net and the physical characteristics. For small values of  $R$ , we have



Physically, the temperature of any point above the characteristic line ( $z = v\theta$ ) has no influence on the temperature at any point on the characteristic. Yet, the finite-difference formula for small  $R$  weights the temperature  $T_i$  heavily in determining  $T_{i+1}$ ; obviously this must be an inaccurate procedure. For large  $R$ , we have



and a similar situation with respect to the influence of  $T_i$  on  $T_{i+1}$ ; the inaccuracy is further enhanced by the fact that physically the temperature  $T_i'$  also has little influence on  $T_{i+1}$ . For  $R = 2$ , the temperatures which affect  $T_{i+1}$  in the finite-difference solution are limited to those which can physically affect  $T_{i+1}$ ; hence, this gives the optimum accuracy.

Based on this simple example, it is possible to formulate some rules for determining a suitable time increment in the more complicated problem of a heat exchanger. This problem has two characteristics (one corresponding to each fluid) and neither is in general a straight line in the  $z-\theta$  plane (that is, the fluid velocity varies). Accordingly the minimum time increment should be based on the side of the heat exchanger which yields the maximum dwell time of the fluid. In addition, although  $R = 2$  gives the optimum accuracy for the simple case of an insulated duct, we will choose  $R = 1$  as being a reasonable compromise between solution accuracy at small values of  $\theta$  and calculation time (that is, number of steps required to cover a given time interval). Therefore, the expression for  $\Delta\theta_{\min}$  is obtained from Equation IV-6 with  $R = 1$ ,

$$\Delta\theta_{\min} = \max \left[ \left( \frac{\Delta z}{L} \right)_r \theta_{d,r}, \left( \frac{\Delta z}{L} \right)_s \theta_{d,s} \right]$$

It should be noted that based on the results of the simple example described here, considerable latitude exists in the choice of  $\Delta \theta$ . Therefore, the terms  $\theta_{d,r}$  and  $\theta_{d,s}$  need only be determined very approximately.

#### Calculation of Maximum Time Increment

Another requirement for accuracy follows from the finite-difference approximation to  $\partial T / \partial \theta$ . If this is to be reasonably accurate, it is essential that  $T$  not change much in one time interval due to normal heat-convection effects; that is,  $\Delta \theta$  should be chosen such that

$$\frac{\Delta T}{T} \ll 1$$

(IV-8)

This requirement can be expressed more quantitatively and used to generate a simple rule for calculating the maximum time step.

The selected procedure involves extrapolating the curves of  $T_{out}$  versus  $\theta$  for each side and estimating the value of  $\Delta \theta$  required to achieve a certain reference  $\Delta T$  in a reasonable number of time steps (say 10). The outlet position of each fluid is chosen, since the outlet conditions are of the major interest. A convenient  $(\Delta T)_{ref}$  to choose for each fluid is the absolute temperature difference between the two fluids at the outlet of the one being considered. Based on these considerations, the expression used to compute  $\Delta \theta_{max}$  is,

$$\Delta \theta_{max} = \min \left[ \Delta \theta_{max,r}, \Delta \theta_{max,s} \right]$$

(IV-9)

where

$$\Delta \theta_{max,r} \equiv 0.1 \left| \frac{(T_r - T_s)_{out,r}}{(\Delta T / \Delta \theta)_{out,r}} \right|$$

(IV-10)

$$\Delta\theta_{max,s} \equiv 0.1 \left| \frac{(T_r - T_s)_{out,s}}{(\Delta T / \Delta\theta)_{out,s}} \right|$$

(IV-11)

The terms  $(\Delta T / \Delta\theta)_{out}$  and  $(T_r - T_s)_{out}$  in these relations are obtained by extrapolating the results of the transient calculations obtained at the previous time. The minimum of the two values is taken so as to insure that the requirement expressed in Equation IV-8 is satisfied for both fluids.

For the case where the r fluid is a single-component condensing fluid and is two-phase at outlet, a different expression must be used to compute  $\Delta\theta_{max,r}$ . This is based on the variation of the exit quality with time and is given by the relation,

$$\Delta\theta_{max,r} \equiv \frac{0.01}{(\Delta x_v / \Delta\theta)_{out,r}}$$

(IV-12)

where a reference quality difference of 0.1 has been selected.

The question arises as to the value of  $\Delta\theta$  to be used if  $\Delta\theta_{max} < \Delta\theta_{min}$ . In this case,  $\Delta\theta = \Delta\theta_{min}$  should be used since the transport effects (represented roughly by the characteristic properties) would appear to be the more important in terms of affecting the accuracy of the solutions.

## APPENDIX V

### DERIVATION OF REDUCED FORMS OF DIFFERENTIAL EQUATIONS

The calculation of the transient performance involves the simultaneous solution of a set of five differential equations, as discussed in the section of this report titled, "Governing Equations". In performing these calculations, it is more convenient to work with a set of two differential equations, one for each fluid; this is done by using a finite-difference representation of the time derivatives,

$$\frac{\partial T}{\partial \theta} = \frac{T - T'}{\Delta \theta}$$

$$\frac{\partial x_v}{\partial \theta} = \frac{x_v - x_v'}{\Delta \theta}$$

and algebraically reducing the original set. In all cases, the reduced form of the differential equation for the s fluid is,

$$\frac{\partial T_s}{\partial \xi} + P_s T_s = Q_{s1} + Q_{s2} T_r \quad (V-1)$$

The corresponding relation for the r fluid is,

$$\frac{\partial T_r}{\partial x} + P_r T_r = Q_{r1} + Q_{r2} T_s \quad (V-2)$$

for cases where the r fluid is single-phase or a wet gas. If the r fluid is a single-component condensing fluid,  $x_{v,r}$  replaces  $T_r$  as the unknown quantity in the two-phase region, and the following expression is used instead of equation V-2:

$$\frac{\partial x_{v,r}}{\partial x} + P_{TP} x_{v,r} = Q_{TP1} + Q_{TP2} T_s$$

(V-3)

This appendix describes the algebraic reduction of the set of differential equations and presents the relations defining the coefficients of the reduced equations. In addition, the expressions relating the three wall temperatures to the fluid temperatures (Equations 28, 29, and 30) are derived, and the coefficients appearing in them are defined.

#### Equation for $T_w$

The desired form of the equation for the separating wall is,

$$T_w = F_{w1} T_i + F_{w2} T_s + F_{w3} T_w' + F_{w, \theta, L} T_r' - F_{w, x, L} \frac{\partial T_r}{\partial x}$$

(V-4)

For the case where the r fluid is not a wet gas, the pertinent relation is given by Equation 9. Using the finite-difference representation of  $\partial T_w / \partial \theta$ , this becomes

$$\frac{\bar{C}_w}{\Delta \theta} (T_w - T_w') = U_{T,r} (T_r - T_w) + U_{T,s} (T_s - T_w)$$

(V-5)

Defining,

$$D_w \equiv \frac{\bar{C}_w}{\Delta \theta} + U_{T,r} + U_{T,s}$$

$$F_{w1} \equiv \frac{U_{T,r}}{D_w}$$

(V-6)



and solving for  $T_w$ , we obtain

$$T_w = F_{w1} T_r + \left( \frac{U_{T,s}}{D_w} \right) T_s + \left( \frac{\bar{C}_w / \Delta \theta}{D_w} \right) T_w' \quad (V-7)$$

When the r fluid is a wet gas, the pertinent relation for the separating wall is Equation 18. In finite-difference form this becomes,

$$\left( \frac{\bar{C}_w}{\Delta \theta} \right) (T_w - T_w') = U_{T,r} (T_r - T_w) + U_{T,s} (T_s - T_w) - \bar{N}_{v,w} \left[ \left( h_{fg,DP} \frac{\partial \omega_r}{\partial T_r} \right) \left( \frac{W_{g,r}}{\Delta \theta} (T_r - T_r') + w_{g,r} \frac{\partial T_r}{\partial x} L_x \right) \right] \quad (V-8)$$

Defining,

$$\lambda_L \equiv h_{fg,DP} \frac{\partial \omega_r}{\partial T_r}$$

$$\lambda_{\theta,L} \equiv \lambda_L \left( \frac{W_{g,r}}{\Delta \theta} \right)$$

$$\lambda_{x,L} \equiv \lambda_L (w_{g,r} L_x)$$

(V-9)

and solving for  $T_w$  gives,

$$T_w = \left[ F_{w1} - \left( \frac{\lambda_{\theta,L} \bar{N}_{v,w}}{D_w} \right) \right] T_r + \left( \frac{U_{T,s}}{D_w} \right) T_s + \left( \frac{\bar{C}_w / \Delta \theta}{D_w} \right) T_w' + \left( \frac{\lambda_{\theta,L} \bar{N}_{v,w}}{D_w} \right) T_r' - \left( \frac{\lambda_{x,L} \bar{N}_{v,w}}{D_w} \right) \frac{\partial T_r}{\partial x} \quad (V-10)$$

Comparison of Equations V-7 and V-10 with Equation V-4 gives the following general definitions of the coefficients,

$$F_{w1}^* \equiv F_{w1} - F_{w,\theta,L}$$

$$F_{w2} \equiv \frac{U_{T,s}}{D_w}$$

$$F_{w3} \equiv \frac{\bar{C}_w / \Delta \theta}{D_w}$$

(V-11)

where

$$F_{w,\theta,L} = F_{w,x,L} = 0$$

(V-12)

when the r fluid is not a wet gas, and

$$F_{w,\theta,L} \equiv \frac{\lambda_{\theta,L} \bar{N}_{v,w}}{D_w}$$

$$F_{w,x,L} \equiv \frac{\lambda_{x,L} \bar{N}_{v,w}}{D_w}$$

(V-13)

when the r fluid is a wet gas.

Equation for  $T_{sw,r}$

The desired form of the equation for the side wall in contact with the r fluid is,

$$T_{sw,r} = F_{sw,r1}^* T_r + F_{sw,r2} T'_{sw,r} + F_{sw,\theta,L} T_r' - F_{sw,x,L} \frac{\partial T_r}{\partial x}$$

(V-14)

For the case where the r fluid is not a wet gas, the pertinent relation is Equation 10. With the finite-difference representation of  $\partial T_{sw,r} / \partial \theta$  this becomes,

$$\left( \frac{\bar{C}_{sw,r}}{\Delta \theta} \right) (T_{sw,r} - T'_{sw,r}) = U_{T,sw,r} (T_r - T_{sw,r})$$

(V-15)

Defining

$$D_{sw,r} \equiv \frac{\bar{C}_{sw,r}}{\Delta \theta} + U_{T,sw,r}$$

$$F_{sw,r1} \equiv \frac{U_{T,sw,r}}{D_{sw,r}}$$

(V-16)

and solving for  $T_{sw,r}$  gives,

$$T_{sw,r} = F_{sw,r1} T_r + \left( \frac{\bar{C}_{sw,r}/\Delta\theta}{D_{sw,r}} \right) T'_{sw,r} \quad (V-17)$$

When the r fluid is a wet gas, the relation is given by Equation 19. Using the finite-difference representation of the time derivatives, we obtain

$$\begin{aligned} \left( \frac{\bar{C}_{sw,r}}{\Delta\theta} \right) (T_{sw,r} - T'_{sw,r}) &= U_{T,sw,r} (T_r - T_{sw,r}) \\ &- (1 - \bar{N}_{v,w}) \left[ \left( h_{fg,DP} \frac{\partial \omega_r}{\partial T_r} \right) \left( \frac{W_{g,r}}{\Delta\theta} (T_r - T'_r) + W_{g,r} \frac{\partial T_r}{\partial x} L_x \right) \right] \end{aligned} \quad (V-18)$$

Using the definitions of  $\lambda_L$ ,  $\lambda_{\theta,L}$ , and  $\lambda_{x,L}$  (Equation V-9) and solving for  $T_{sw,r}$  gives,

$$\begin{aligned} T_{sw,r} &= \left[ F_{sw,r1} - \left( \frac{\lambda_{\theta,L}(1 - \bar{N}_{v,w})}{D_{sw,r}} \right) \right] T_r + \left( \frac{\bar{C}_{sw,r}/\Delta\theta}{D_{sw,r}} \right) T'_{sw,r} \\ &+ \left( \frac{\lambda_{\theta,L}(1 - \bar{N}_{v,w})}{D_{sw,r}} \right) T'_r - \left( \frac{\lambda_{x,L}(1 - \bar{N}_{v,w})}{D_{sw,r}} \right) \frac{\partial T_r}{\partial x} \end{aligned} \quad (V-19)$$

Comparing Equations V-17 and V-19 with Equation V-14, we arrive at the following general definitions of the coefficients,

$$\begin{aligned} F_{sw,r1}^* &\equiv F_{sw,r1} - F_{sw,\theta,L} \\ F_{sw,r2} &\equiv \frac{(\bar{C}_{sw,r}/\Delta\theta)}{D_{sw,r}} \end{aligned} \quad (V-20)$$

where

$$F_{sw,\theta,L} = F_{sw,x,L} = 0 \quad (V-21)$$

when the r fluid is not a wet gas, and

$$F_{sw,\theta,L} \equiv \frac{\lambda_{\theta,L}(1-\bar{N}_{v,w})}{D_{sw,r}}$$

$$F_{sw,x,L} \equiv \frac{\lambda_{x,L}(1-\bar{N}_{v,w})}{D_{sw,r}}$$

(V-22)

in cases where the r fluid is a wet gas.

#### Equation for $T_{sw,s}$

The desired form of the equation for the side wall in contact with the s fluid is,

$$T_{sw,s} = F_{sws1} T_s + F_{sws2} T'_{sw,s}$$

(V-23)

In all cases, the pertinent relation is Equation 11. Using the finite-difference representation for  $\partial T_{sw,s} / \partial \theta$  gives,

$$\left( \frac{\bar{C}_{sw,s}}{\Delta \theta} \right) (T_{sw,s} - T'_{sw,s}) = U_{T,sw,s} (T_s - T_{sw,s})$$

(V-24)

Defining,

$$D_{sw,s} \equiv \frac{\bar{C}_{sw,s}}{\Delta \theta} + U_{T,sw,s}$$

(V-25)

and solving for  $T_{sw,s}$  we obtain

$$T_{sw,s} = \left( \frac{U_{T,s}}{D_{sw,s}} \right) T_s + \left( \frac{\bar{C}_{sw,s} / \Delta \theta}{D_{sw,s}} \right) T'_{sw,s}$$

(V-26)

Comparing Equation V-26 with V-23 gives the following definitions of the coefficients.

$$\begin{aligned} F_{sws1} &\equiv \frac{U_{T,s}}{D_{sw,s}} \\ F_{sws2} &\equiv \frac{(\bar{c}_{sw,s}/\Delta\theta)}{D_{sw,s}} \end{aligned} \quad (V-27)$$

### Equations for r Fluid

#### Temperature Equation

For cases where the r fluid is single phase (including in this category the single-phase regions of a single-component condensing fluid) or a wet gas, the differential equations for  $T_r$  (Equations 7 and 17) can be written in the following general form:

$$(W_r^* c_{p,r}^*) \frac{\partial T_r}{\partial \theta} = -(w_r^* c_{p,r}^*) \frac{\partial T_r}{\partial x} L_x + U_{T,r} (T_w - T_r) + U_{T,sw,r} (T_{sw,r} - T_r) \quad (V-28)$$

where

$$\begin{aligned} W_r^* &\equiv W_r \\ w_r^* &\equiv w_r \\ c_{p,r}^* &\equiv c_{p,r} \end{aligned} \quad (V-29)$$

if the fluid is not a wet gas, or

$$\begin{aligned} W_r^* &\equiv W_{g,r} \\ w_r^* &\equiv w_{g,r} \\ c_{p,r}^* &\equiv c_{p,g,r} + \omega_r c_{p,v,r} \end{aligned} \quad (V-30)$$

if the fluid is a wet gas.

Using the finite-difference representation of  $\partial T_r / \partial \theta$  and substituting Equations V-4 and V-14 for  $T_w$  and  $T_{sw,r}$  in Equation V-28 gives the following expression:

$$(w_r^* c_{p,r}^* L_x + \alpha_{x,L}) \frac{\partial T_r}{\partial x} + D_r T_r = E_r' + U_{T,r} F_{w2} T_s \quad (V-31)$$

where,

$$\alpha_{x,L} \equiv U_{T,r} F_{w,x,L} + U_{T,sw,r} F_{sw,x,L}$$

$$D_r \equiv \frac{W_r^* c_{p,r}^*}{\Delta \theta} + U_{T,r} (1 - F_{w1}) + U_{T,sw,r} F_{swr2} + \alpha_{\theta,L}$$

$$\alpha_{\theta,L} \equiv U_{T,r} F_{w,\theta,L} + U_{T,sw,r} F_{sw,\theta,L}$$

$$E_r' \equiv \left( \frac{W_r^* c_{p,r}^*}{\Delta \theta} + \alpha_{\theta,L} \right) T_r' + (U_{T,r} F_{w3}) T_w' + (U_{T,sw,r} F_{swr2}) T_{sw,r}' \quad (V-32)$$

Comparing Equation V-31 with V-2, we arrive at the following definitions of the coefficients,

$$\begin{aligned} F_r &\equiv \frac{D_r}{w_r^* c_{p,r}^* L_x + \alpha_{x,L}} \\ Q_{r1} &\equiv \frac{E_r'}{w_r^* c_{p,r}^* L_x + \alpha_{x,L}} \\ Q_{r2} &\equiv \frac{U_{T,r} F_{w2}}{w_r^* c_{p,r}^* L_x + \alpha_{x,L}} \end{aligned} \quad (V-33)$$

#### Vapor-Quality Equation

For the case where the r fluid is a single-component condensing fluid in the two-phase region, the pertinent relation is Equation 16. Using the finite-difference representation of  $\partial x_{v,r} / \partial \theta$ , this relation becomes

$$\left( \frac{W_r h_{fg,sat}}{\Delta \theta} \right) (x_{v,r} - x_{v,r}') = - (w_r h_{fg,sat}) \frac{\partial x_{v,r}}{\partial x} L_x + U_{T,r} (T_w - T_r) + U_{T,sw,r} (T_{sw,r} - T_r) \quad (V-34)$$

Substituting Equations V-4 and V-14 (with  $F_{w,\theta,L} = F_{w,x,L} = F_{sw,\theta,L} = F_{sw,x,L} = 0$ ) for  $T_w$  and  $T_{sw,r}$  and  $T_r = T_{sat}$  in Equation V-34 gives,

$$(w_r h_{fg,sat} L_x) \frac{\partial x_{v,r}}{\partial x} + \left( \frac{W_r h_{fg,sat}}{\Delta \theta} \right) x_{v,r} = E'_{TP} - \left[ U_{T,r}(1-F_{w1}) + U_{T,sw,r} F_{swr2} \right] T_{sat} + (U_{T,r} F_{w2}) T_s \quad (V-35)$$

where

$$E'_{TP} \equiv \left( \frac{W_r h_{fg,sat}}{\Delta \theta} \right) x'_{v,r} + (U_{T,r} F_{w3}) T'_w + (U_{T,sw,r} F_{swr2}) T'_{sw,r} \quad (V-36)$$

Comparison of Equation V-35 with V-3 gives the following definitions of the coefficients,

$$P_{TP} \equiv \frac{(W_r h_{fg,sat} / \Delta \theta)}{w_r h_{fg,sat} L_x}$$

$$Q_{TP1} \equiv \frac{E'_{TP} - [U_{T,r}(1-F_{w1}) + U_{T,sw,r} F_{swr2}] T_{sat}}{w_r h_{fg,sat} L_x}$$

$$Q_{TP2} \equiv \frac{U_{T,r} F_{w2}}{w_r h_{fg,sat} L_x} \quad (V-37)$$

#### Equation for s Fluid

In all cases, the pertinent relation for  $T_s$  is Equation 8. Using the finite-difference representation of  $\partial T_s / \partial \theta$  and substituting Equations V-4 and V-23 for  $T_w$  and  $T_{sw,s}$  in Equation 8, we obtain the following expression:

$$(w_s c_{p,s} L_s) \frac{\partial T_s}{\partial \xi} + D_s T_s = E'_s + (U_{T,s} F_{w1}^*) T_r - (U_{T,s} F_{w,x,L}) \frac{\partial T_r}{\partial x} \quad (V-38)$$

where

$$D_s \equiv \frac{W_s c_{p,s}}{\Delta \theta} + U_{T,s}(1 - F_{w2}) + U_{T,sw,s} F_{sws2}$$

$$E'_s \equiv \left( \frac{W_s c_{p,s}}{\Delta \theta} \right) T'_s + U_{T,s} (F_{w3} T'_w + F_{w,0,L} T'_r) + U_{T,sw,s} F_{sws2} T'_{sw,s} \quad (V-39)$$

Using Equation V-2 to substitute for  $\partial T_r / \partial x$  in Equation V-38, gives

$$(w_s c_{p,s} L_s) \frac{\partial T_s}{\partial \xi} + (D_s + U_{T,s} F_{w,x,L} Q_{r2}) T_s = (E'_s - U_{T,s} F_{w,x,L} Q_{r1}) + U_{T,s} (F_{w1}^* + F_{w,x,L} P_r) T_r \quad (V-40)$$

Comparison of this relation with Equation V-1 yields the following definitions of the coefficients,

$$P_s \equiv \frac{D_s + U_{T,s} F_{w,x,L} Q_{r2}}{w_s c_{p,s} L_s}$$

$$Q_{s1} \equiv \frac{E'_s - U_{T,s} F_{w,x,L} Q_{r1}}{w_s c_{p,s} L_s}$$

$$Q_{s2} \equiv \frac{U_{T,s} (F_{w1}^* + F_{w,x,L} P_r)}{w_s c_{p,s} L_s}$$

(V-41)